Forest volume estimation and yield prediction

Vol. 1 - Volume estimation

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Mr. F. Cailliez acknowledges with thanks the assistance provided by his colleagues in writing this manual, namely J. Bouchon, P. Duplat, F. Guinaudeau, and N. Ogaya. Thanks also go to Miss C. Gueguen who did the drawings and some typing in French and English.

FOREWORD

There is probably little argument among forest managers that the ability to estimate the volume of trees and stands and to predict what the forest will produce, on different sites, in response to particular types of silvicultural treatment, is central to all rational planning processes connected with forestry. There is, however, a considerable diversity of opinions over what constitutes "yield", and how it may be estimated and projected into the future.

This manual is an attempt to codify current practices in the field of tree and stand volume estimation and forest yield prediction in a way that is practicable and useful to the person who is charged with the responsibility of producing volume estimations and yield forecasts, but perhaps has not had the benefit of extensive experience in this field.

It must be appreciated, however, that this is a field of human endeavour that is currently in a state of rapid evolution, especially with regard to forests growing in tropical environments. Consequently, all that is said in this manual must be regarded as provisional and subject to future refinement for particular situations that can arise, or new techniques that can be developed, whilst other techniques may exist which are not referred to in this text and which may be superior for particular purposes.

Thus it is not a manual in the true sense; it is rather a set of guidelines for the choice of procedure combined with more precise instructions concerning calculation technique for some specified cases.

The manual is done with special reference to the tropics and applies to natural as well as man made forests. Because of the great difficulties in assessing growth and yield of natural mixed and uneven aged forests, the methods given to construct growth models, however, mainly apply to even aged forests. For mixed forests no specific instructions are given but rather some examples of possible ways of dealing with the problem.

The manual consists of two volumes. The first volume describes techniques of measuring trees and the assessment of volume of trees and stands, and the second volume deals with growth and yield prediction. Descriptions of statistical and mathematical techniques, selected statistical tables, blank copies of calculation and data recording forms and an annotated bibliography are included in a series of appendices.

Volume I of the manual has been written by Francis Cailliez, Centre Technique Forestier Tropical (CTFT), Nogent-sur-Marne, France, and Volume II by Denis Alder, Commonwealth Forestry Institute (CFI), Oxford, Great Britain, who also compiled the appendies. The work of the two authors has been coordinated by Jöran Fries, Swedish University of Agricultural Sciences, Uppsala, Sweden. The work was formulated and guided by Jean-Paul Lanly and Karn Deo Singh of the Forest Resources Division of FAO. Jean Clement (CTFT) was associated at the initial stage of the study.

The first draft of the manual was presented at the meeting of the IUFRO Subject Group S4.01 (Mensuration, Growth and Yield) held in Oxford in September 1979, and was discussed for one full day in detail. Among the participants there were tropical forest mensurationists especially invited by FAO to make a thorough and critical review of the contents of the manual. In addition, the manual was also sent to a number of specialists for comments. Based on these remarks, a revised version of the manual was prepared by the authors concerned.

This manual, being the first of its kind in the field of tropical forestry, has considerable scope for further improvements and additions. Particularly in the case of mixed uneven aged stands further complementary studies are immediately needed. All suggestions in this respect will be very much appreciated.

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C O N T E N T S

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0 INTRODUCTION

How to measure the volume of trees and of stands ? The first part of the manual intends to answer this question.

It is, in effect, worth treating this subject for two essential reasons :

. the problem in itself is neither as simple, nor as clearly posed as it seems ; it is necessary to define as precisely as possible the nature of the required volume : is one interested in the ligneous biomass, in the volume of the large stems, in the volume of sawable timber, etc... ?

. once the required volume or volumes have been specified, the way of measuring them has to be defined ; in this field, forest practices are very old and varied and it is important to try to unify them to be able to make valid comparisons between estimates made by different persons in different countries.

Being a manual, the accent has been put on the most robust methods; sophisticated techniques which can only be applied by well equipped research institutes are not mentioned (utilization of expensive dendrometers, measurement of volumes on aerial photographs, etc...).

Furthermore, this manual is addressed mainly to foresters of tropical countries, where the major problems concern the utilization of the wood as fuel and as prime material for the supply of sawmills and of veneer units or for the production of pulp. This is why the other uses of the forest (harvest of minor products such as cork, utilization of fodder of the forest, etc...) which pose specific problems as regards the measurement of these products, have not been considered.

After having defined the most important types of volume, a description is given of the procedures to be followed to collect the data and for the calculations to be applied.

This first part of the manual can be considered as an introduction to dendrometry and could as well be placed in a manual on forest inventory ; it can be read easily by every person having to work in this field.

1 THE DIFFERENT VOLUMES THAT CAN BE DEFINED IN A TREE

The volume one is talking about always has to be defined very precisely. An answer has therefore to be given to the following two questions :

- What is the physical object concerned ?
- In what part of this object is one interested ?

Furthermore, it is advisable to specify how the volume of this object was calculated ; this question arises because the real volume is seldom known exactly (it would be the volume of water which the object displaces when immersed in a vat). The procedures for the estimation of this exact volume are described in § 23.

Let us return, for the moment, to the two questions.

11 WHAT IS THE PHYSICAL OBJECT CONCERNED ?

It can be :
- the stem : *The part of the tree which is followed going from the foot of the tree to the terminal bud. For ramified trees one considers conventionally that the terminal bud is the most elevated bud.*
- the branches
- the roots
- the tree : *Stem + branches + roots*

Specify if the bark is included or not.

12 IN WHAT PART OF THIS OBJECT IS ONE INTERESTED

The limits of the object are a lower crosscut (at the larger end) and an upper crosscut (at the smaller end). Each of these crosscuts can be defined in several ways.

121 Dimension crosscuts

The following are three examples of upper crosscuts of this type :

- *the 0 cm diameter crosscut means that the limit is the physical extremity of the stem or of the branch. One then speaks of* "total" *volume.*
- *the 7 cm diameter crosscut is the most often used one to make a limit with the twigs, which are very numerous, difficult to measure and of little interest. It is the upper crosscut of the volume of* "big wood".

- *other crosscuts are possible : the 5 cm diameter crosscut is, for example, often accepted as the upper crosscut of the pulpwood volume.*

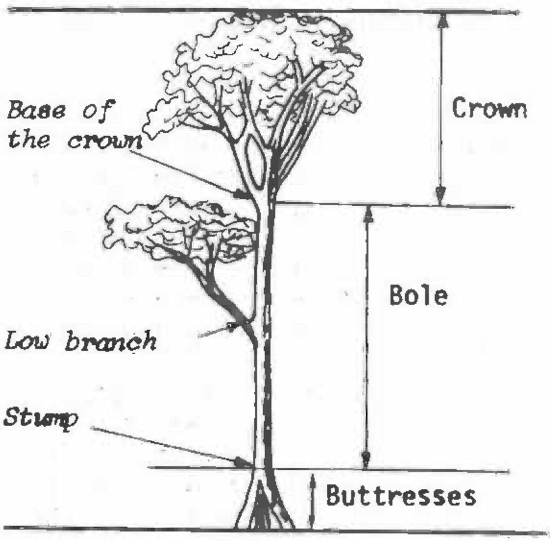
122 Form crosscuts - Examples

122.1 The stump : *can be defined as the base of the part of the stem which is extracted from the forest under optimal exploitation conditions (losing the least possible utilizable volume). For trees without buttresses, this level is at a height from the ground of between 10 and 50 cm in general ; if this level is not specified, it is presumed to be at a distance from the ground equal to a hundredth of the total height of the tree. For trees with buttresses or with aerial roots, the stump is the top of the buttresses or of the roots (a level which is, in general, higher than the felling height).*

122.2 The base of the crown : *it is the place where the stem clearly ramifies.*

These two crosscuts allow one to define :

the bole	: *the part of the stem situated between the stump and the base of the crown.*
the low branches	: *the branches inserted on the bole.*
the crown	: *the part of the stem situated above the base of the crown + the branches inserted above the base of the crown.*



123 Quality crosscuts

One mentions for example slicing crosscut, sawing crosscut, post or pole crosscut, etc... These notions are evidently rather delicate to appreciate because they comprise notions of form and of dimension and are closely connected to technological and commercial practices of the moment.

13 SOME EXAMPLES OF GROSS VOLUMES

The foregoing shows that for a given tree one can define an almost infinity of volumes. Here are some examples, given in order of increasing complexity of their measurement. The first three are the most used.

Physical object	Lower crosscut (at the larger end)	Upper crosscut (at the smaller end)	Name of volume
Bole	Stump	Base of the crown	Bole volume
Stem	Stump	Crosscut D = 7cm	Big wood stem volume
Stem	Stump	Physical extremity	Total stem volume
Stem + low branches	For the stem : stump	Crosscut D = 7 cm for the stem and each low branch	Big wood volume stem + low branches
Stem + branches	Stump	Physical extremity of the stem and of each branch	Total volume above ground
Stem + branches	Stump	Crosscut D = 7cm for the stem and each branch	Big wood volume above ground
Branches	For each branch : insertion on the stem or on the branch on which it is inserted.	Physical extremity of each branch	Total branch volume
Branches	- idem -	Crosscut D = 7cm for each branch	Big wood branch volume
Crown	For the stem crosscut D = 7cm for each branch: insertion on the stem.	Physical extremity of the stem and of each branch	Total crown volume (lower crosscut D=7cm for the stem)
Stem + branches	Ground level	Physical extremity of the stem and of each branch	Total ligneous biomass above ground
Tree	Physical extremity of stem, branches, roots		Total tree ligneous biomass

Specify each time if it is volume with or whithout bark and indicate how this volume has been calculated.

14 CONCERNING USABLE VOLUMES

Volumes cited above are gross volumes. How to estimate the corresponding usable volumes would require lengthy explanations which cannot be tackled in this manual. The subject is indeed difficult and is far from being solved.

The difficulties are of several types :

- the current and future uses of the wood have to be known (veneer, sliced wood, sawn wood, telegraphic poles, pulp wood, fuel wood, chip wood,...)

- for each use, the chain of transformations to which the wood will be submitted has to be known in detail (felling, skidding and transportation systems, industrial processing,...).

- the different constraints imposed by these transformations have to be expressed in terms of measurable data (length, diameter, cylindricity, heart eccentricity, stem bending, admissible defects...).

The procedure to be followed in order to convert gross volumes into usable volumes is therefore very much subordinate to local conditions and available resources. We can only give a few ideas and send the reader back to specialized handbooks.

- Data collected by foresters during inventories (observations on standing or felled trees) can only provide volumes presumed suitable for such and such use. Inquiries among logging companies and processing industries are essential to determine the exact transformation coefficients from gross volume to used volume. An example will be given in paragraph 41,

- even when a volume estimation is undertaken with a well defined purpose (for instance, pulpmill supply), data should be collected in order to be able to estimate other volumes because the final destination of wood and/or loggers and manufacturers requirements might change in future,

- give priority to gross volume estimation and consider usuable volume estimation as a specialized task.

2 DIRECT MEASUREMENT OF THE VOLUME OF A TREE

According to the type of the required volume, the measurements will be more or less numerous. As the various parts of the tree (stem, branches) never are solids of a perfectly known geometrical form, such as cylinders, cones etc..., the principle is to measure on each of them the diameter at different heights and to calculate the volume from these measurements ; of course, this volume will be the more exact as the number of measured diameters will be large. It is obvious that these measurements are easier and more accurate on felled than on standing trees, which explains the lay out of this paragraph.

§ 21 Measurements on standing trees } *Calculation of the volume*
§ 22 Measurements on felled trees } *based on these measurements:* §23

21 MEASUREMENTS ON STANDING TREES

211 Measurements of size (diameter or girth)

The size of a tree is traditionally described by the following values : reference diameter, reference circumference, basal area. One measures the diameter or the circumference and the basal area is deducted by the formula corresponding to the circle :

$$\text{Basal area} = \frac{\pi}{4}\,(\text{reference diameter})^2$$

$$= \frac{1}{4\pi}\,(\text{reference circumference})^2$$

The basal area is thus a conventional value which gives an approximation of the area of the reference section. The knowledge of the exact value of the area of this section is indeed practically impossible on a standing tree and, on a felled tree, it requires the use of a planimeter.

211.1 Definition of the reference diameter and of the reference circumference

Between all the diameters and all the circumferences that can be measured, the reference diameter *and the* reference circumference *play an essential role.*

On a standing tree, this diameter (or this circumference) is measured :

- at 1.30 m from the ground (4'3") *for trees without buttresses or with buttresses or aerial roots less than 1 m. high. The reference diameter is then, traditionally, called* diameter at breast height . *It is recommended to avoid this ambiguous expression and to take care that the height of measurement does not depend upon the height of the operator.*

- at 30 cm above the end of the buttress or the aerial roots *if these are higher than 1 meter.*

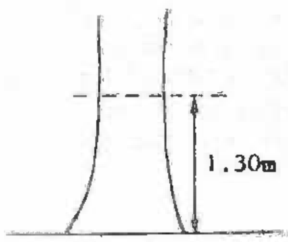
When the height from the ground is not equal to 1.3 m, it should be recorded.

The following page illustrates some cases which occur in practice for the definition of the reference diameter.

REFERENCE DIAMETER

Flat terrain

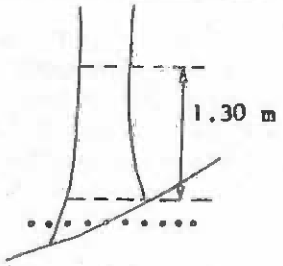
Straight tree without buttresses or with buttresses less than 1 meter or with aerial roots less than 1 meter.



Sloping terrain

Vertical tree

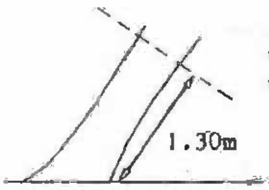
As a rule, the base of the tree is the level marked ... (location of the seed). For practical reasons the measurement is taken at 1.30 m at the uphill side.



Leaning trees

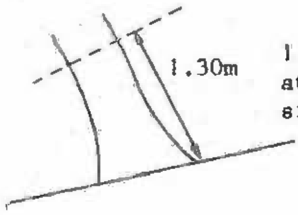
The 1.30 m length has to be measured parallel to the tree, not vertically. The measured section has to be perpendicular to the axis of the tree, not horizontal.

Flat terrain



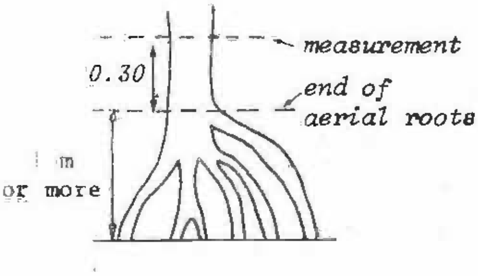
1.30 m measured on the side where tree is leaning

Sloping terrain



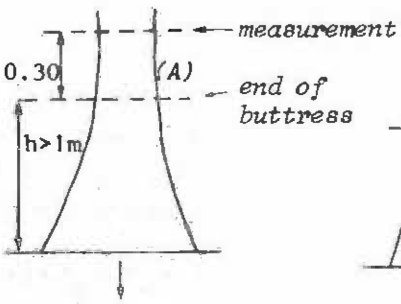
1.30 m measured at up hill side

Trees with aerial roots higher than 1 m.



Trees with buttresses higher than 1 meter

For a good estimate of level (A) view the tree from a distance

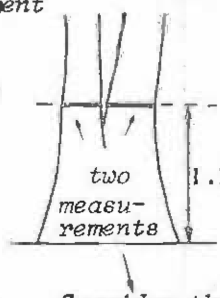


in general, h *is smaller than 6 m.*

Forked trees

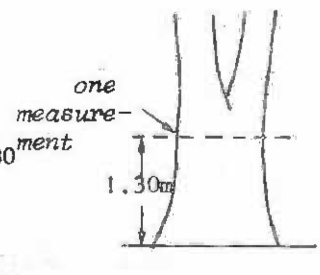
Bottom of the fork

Less than 1.30 m

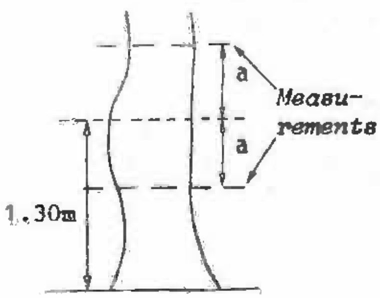


Consider there are two trees.

Higher than 1.30 m



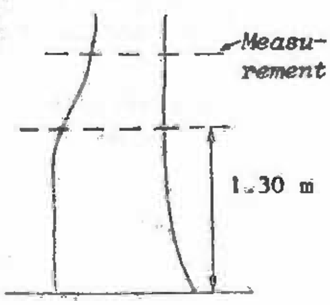
Anomaly at 1.30 m (knot, swelling, deformation...)



The measuremsnts have to be taken outside the deformed part.

Do, if possible, 2 measurements at equal distances from the 1.30 m level and take average.

But it can happen that only one measurement is possible.



Remarks :

- *If diameter has to be remeasured in the future for increment, the level of measurement has to be materialized (painted mark,...) Such a mark can induce a reaction of the tree ; it is thus advisable to put the mark at a fixed distance (for instance 10 cm) from the measurement level and to record the height of the mark in case it should disappear.*
- *In inventory or permanent sample plot mensurational work, reference diameter is in general measured only for trees of a minimum size. In most cases, reference diameter is measured if it is more than 5 cm (on smaller trees, height is measured) but on studies focused on regeneration, diameter measurement is of interest and requires special instruments (mini calipers).*

211.2 Practice of diameter measurements on standing trees

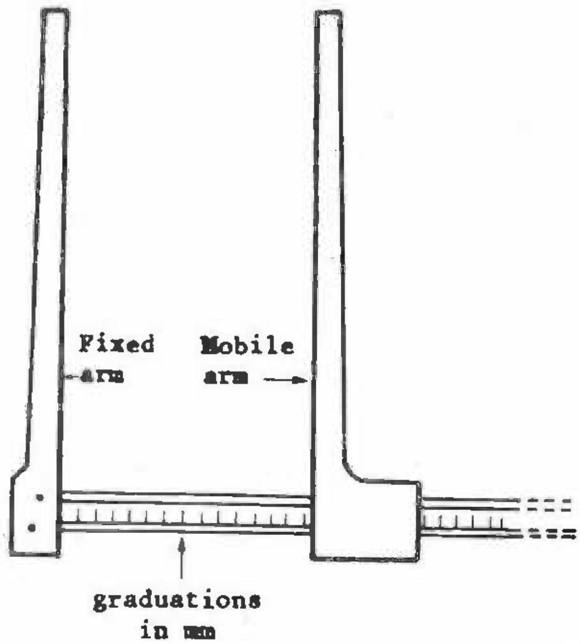
The following scheme indicates the lay-out of the paragraph.

The height of measurement is manually accessible (eventually by climbing or with poles)

- YES
  - Usual caliper - Finnish caliper
  - Tape
  - For measurement of big diameters : ruler
- NO
  - Commercial instruments for optical measurements : 2 examples
    - Wheeler pentaprism
    - Bitterlich relascope

211.21 Diameter measurement with a caliper

211.211 The usual caliper

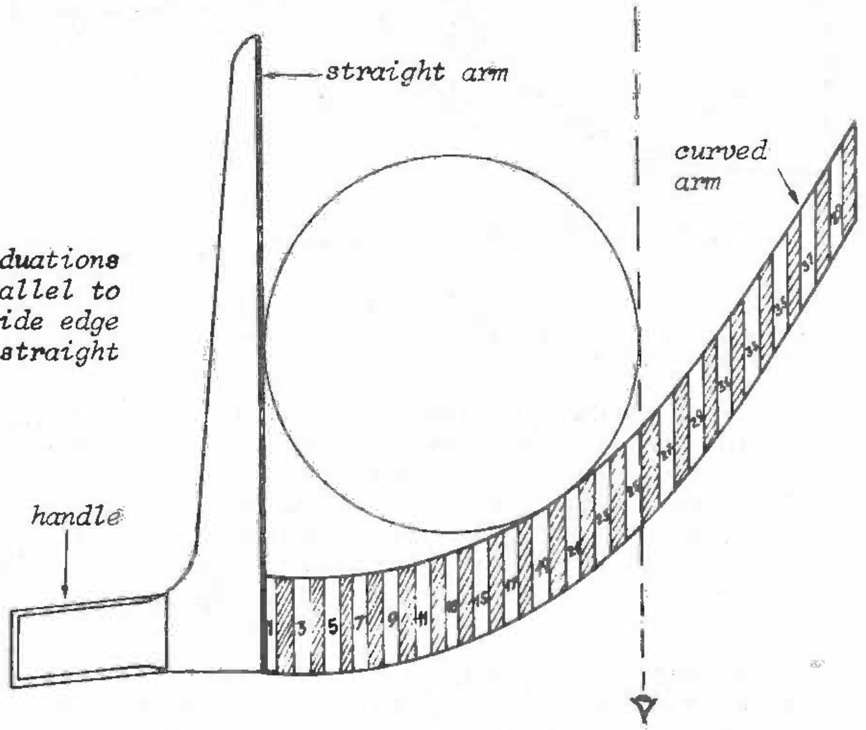


- *Do prefer a metallic caliper to a wooden caliper (climatic stability - easy to clean).*
- *Hold horizontally.*
- *Do not press the arms too much against the tree (soft bark).*
- *Verify frequently the parallelism of the arms.*
- *Take at least one measurement, without choosing the direction. For a better precision or for a flat tree, do a second measurement in the perpendicular direction and take the arithmetic average.*
- *Carry out the measurement with the maximum precision allowed by the graduation (in general to the nearest cm, if possible to the nearest mm).*

Various improvements can be made to the instrument, of which the above drawing shows the most simple type : additional graduations (girth, basal area), adjustment by a screw of the final position of the mobile arm, movement on rollers of the mobile arm, addition of a system of automatic registration of the measurement on punched paper tape or mini cassette,...

211.212 The Finnish caliper

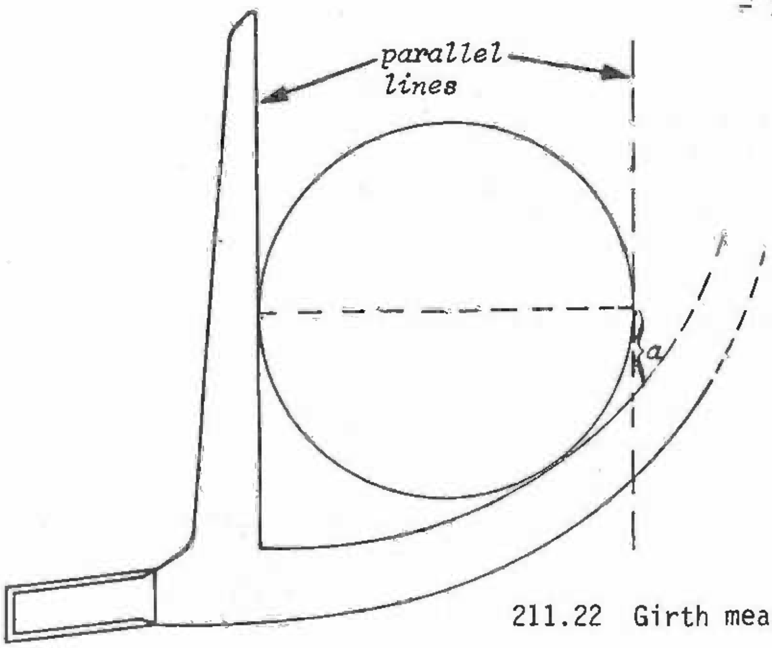
The graduations are parallel to the inside edge of the straight arm.



Grasp the handle with left hand (the left arm should be stretched out as far as possible), apply the caliper against the tree, orthogonal to stem axis. The diameter is obtained by sighting parallel to the graduation marks.

Advantages on the usual caliper :

- *no movable part,*
- *when fixed on telescopic poles, the caliper allows to measure diameters up to approximately 8 m from the ground, and even up to a dozen meters if binoculars are used for the reading,*
- *instrument easily selfmade with plywood (7 layer, 9 mm thick) ; graduate both faces to be used with left or right hand ; varnish the whole instrument.*



The curved form of the graduated arm is such that the distance a does not depend on tree size, which guarantees the same degree of accuracy for trees of different sizes (a = 5.5 cm in the instrument of previous page).

211.22 Girth measurement with a tape

The use of a tape is indispensable for large trees because the caliper is impracticable. Also for small trees, a tape is preferable to a caliper :

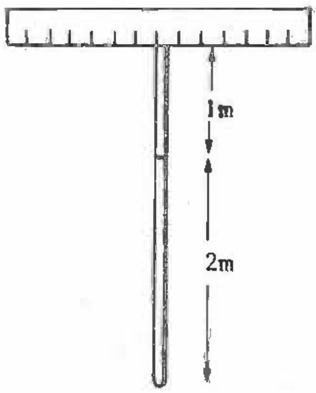
- the tape estimates a size called girth (it is in fact the perimeter of the convex hull of the section), the definition of which is not ambiguous whereas there exists an infinity of diameters. *By reference to a circle, the quotient of the measured girth by π is taken as the diameter (certain tapes comprise a diameter graduation). A mathematical property adds a supplementary justification to this practice : the measured girth divided by π is equal to the average of the infinity of diameters that could be measured with a caliper.*
- measurements with a tape are more reliable than measurements with a caliper : *the tape, provided that it does not extend, is stronger and the risk for compression of the bark is less than with a caliper. It is essentially for this reason that one can hear say that the diameter measured with a tape is systematically larger than the diameter measured with a caliper and it is to correct for this bias that the french standard for example defines the girth at 1.50 m as the reference for the size of a tree. This definition is not to be recommended in order to unify and also because detailed studies, practical as well as theoretical, have shown that the difference between* $D_{1.30m}$ *and* $\frac{1}{\pi} C_{1.30m}$ *is, in general, small and does not have a really systematic character.*
- *The main point is to hold the tape in a plane perpendicular to the stem axis, after having removed lianas, mosses,... (but care must be taken not to remove bark inadvertently). Prefer tapes provided with a hook at the extremity to fix in the bark, which allows a single person to measure a large tree. Linen tapes stretch and wear. Metal tapes are better but kink. With recent materials such as fiberglass, these disadvantages disappear.*
- *Carry out the measurement with the maximum precision allowed by the graduation ; in general to the nearest cm (82.4 → 82 ; 82.6 → 83), if possible to the nearest mm. However, for rapid measurements under difficult conditions and with unskilled labour, whole unit measure (82.4 → 82 ; 82.6 → 82), though biased, may be more accurate because of the reduced risk of misunderstanding.*

211.23 Method of the ruler for diameter measurement at small heights (5 to 6 m maximum)

211.231 Construction of the ruler

a) *Take a board of 150 cm × 10 cm × 1 cm and paint white.*

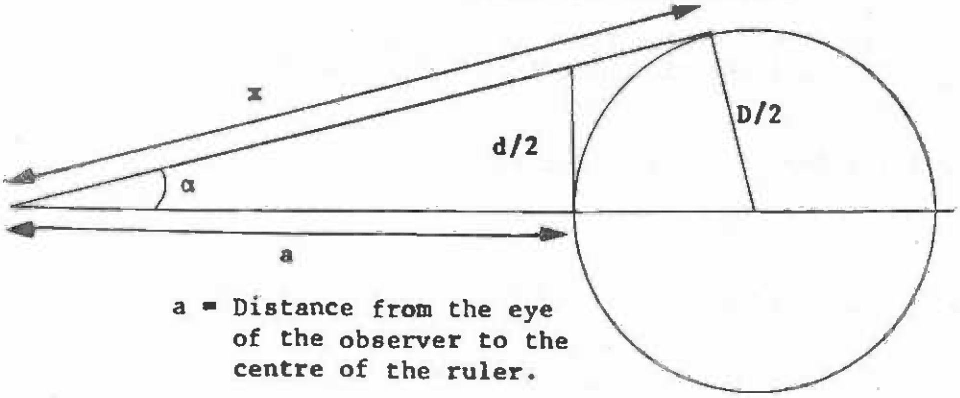
b) *Attach in the middle a rod of 1 m long and on this rod a detachable handle of 2 m long.*



c) *Mark with black paint the limits and the numbers of the classes as given below.*

Classes	Lower limits of the classes	
	Exact limits D (cm)	Positions of limits on ruler d (cm)
2	15	14.9
3	25	24.7
4	35	34.4
5	45	44.0
6	55	53.6
7	65	63.0
8	75	72.4
9	85	81.7
10	95	90.9
11	105	99.9
12	115	108.9
13	125	117.9
14	135	126.7
15	145	135.5

<u>Justification</u> : *to avoid parallax errors, the class limits on the ruler are corrected :*



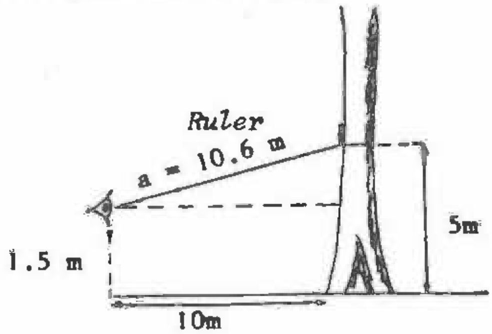
a = Distance from the eye of the observer to the centre of the ruler.

One can prove easily that :

$$d = \frac{D}{\sqrt{1 + \frac{D}{a}}}$$

The operating method which follows supposes that the observer is at a horizontal distance of 10 m from the tree. If the ruler is at his eye level, a = 10 m *If not, the maximum height at which the ruler can be placed being approximately 5 m and admitting that the terrain is flat and that the eye of the observer is at 1.5 m from the ground, the distance will then be :*

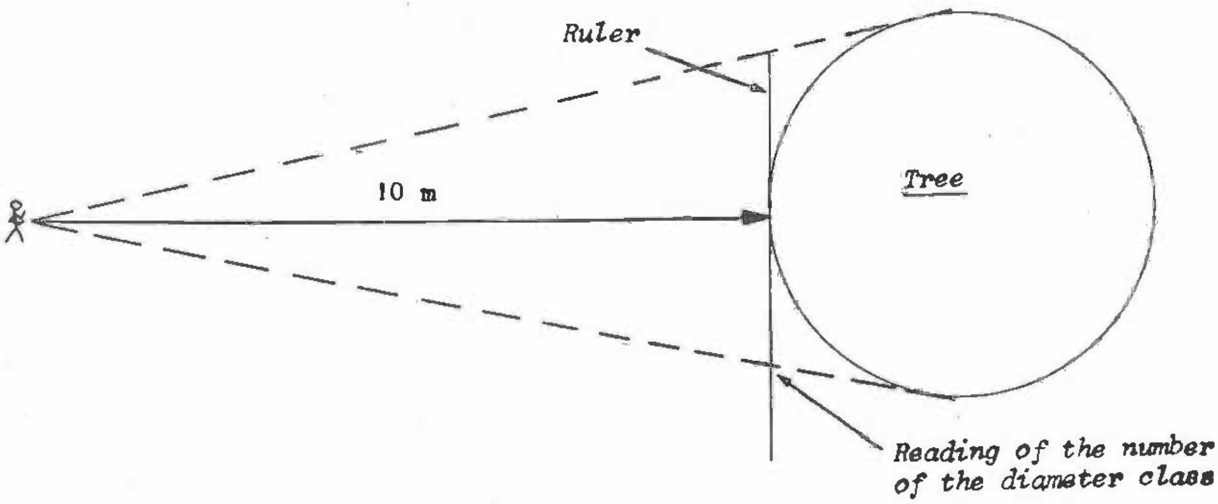
$$\sqrt{10^2 + (5 - 1.5)^2} = 10.6 \text{ m}$$



One can therefore estimate that a *is equivalent to 10.3 m on an average. The corrected diameters* d *which are to be marked on the ruler have been calculated with this value.*

211.232 Operating method

- *The observer places himself at a horizontal distance of 10 m from the side of the tree.*



- *A helper places the ruler against the tree at the height of measurement. It is important that the ruler should be perpendicular on the line of sight. The left edge of the ruler has to be in a line with the left edge of the stem in relation to the observer.*
- *Read the number of the diameter class on the right hand part of the ruler.*

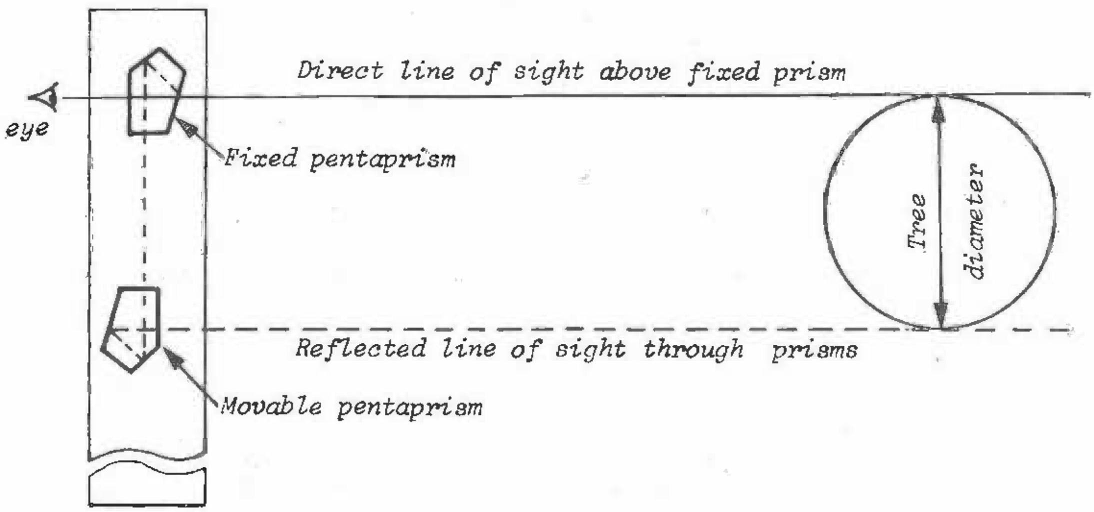
211.24 The Wheeler pentaprism caliper

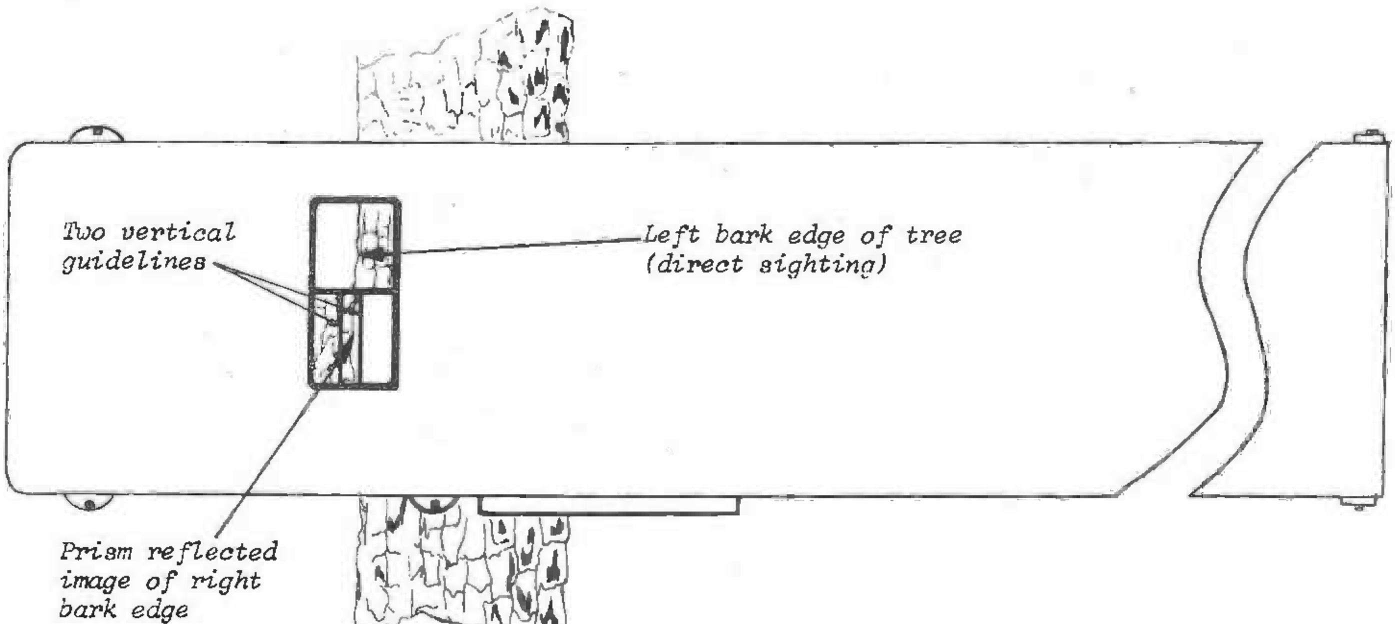
This optical instrument has two advantages :

- *the observer can stand at any distance from the tree and this distance has not to be known,*
- *it allows diameter measurements at any height.*

But it requires a good visibility : a strong contrast between tree and background is necessary.

Diagram showing pathways taken by the sightings through the instrument





Hold the caliper 8 or 10 cm in front of the eye with the graduated scale up. Look into and through the viewing slot. Most operators keep both eyes open.

Through the upper part of the slot, the left bark of the tree is seen directly. In the lower part, appears the right bark edge reflected through the two prisms. Slide the movable prism with the right hand until the right bark edge reflection is brought into direct vertical alignment with the left bark edge, midway between the two vertical guidelines. Read the diameter on the scale.

The instrument exists with 3 lengths :

44 cm → maximum diameter 36 cm

69 cm → maximum diameter 62 cm

95 cm → maximum diameter 86 cm.

To check the caliper for accuracy, measure a target of known width and adjust the pointer position to get the correct value. Verify also that the measurement does not depend upon the distance (some instruments are defective).

211.25 Diameter measurement with the Bitterlich Relascope

The Bitterlich relascope *is an instrument quite universally used by foresters, which permits the following principal measurements :*

a) diameter of the tree at any height
b) tree height
c) basal area of stand
d) certain horizontal lengths
e) slope of a terrain.

Its description, the principle of its functioning , and its handling for the measurements c) d) e) *, are not given ; the following are the directions for use of the instrument (of the wide scale model, more adapted to the measurement of large trees than the narrow scale model) for the measurement of diameters at various heights of the bole.*

Diameter measurements with the wide scale Bitterlich Relascope for the calculation of a bole volume

1/ *Stand at a horizontal distance* D *from the centre of the tree equal to at least 2/3 of its height (* D *can be equal to 4, 6, 8, 10, 12, 14, 16, 18, 20 meters).*

2/ *The bole must be seen completely. Clear vegetation if necessary.*

3/ *Measure height* h_c *of buttress, reference diameter (with a tape or a ruler), and bark thickness.*

4/ H *being the first entire height (read on the scale corresponding to the distance* D*) situated above the end of the buttress, measure :*

- *diameter at end of buttress*
- *height* h *between* H *and end of buttress*
- *diameter at heights* H, H+1, H+3, H+5, etc...

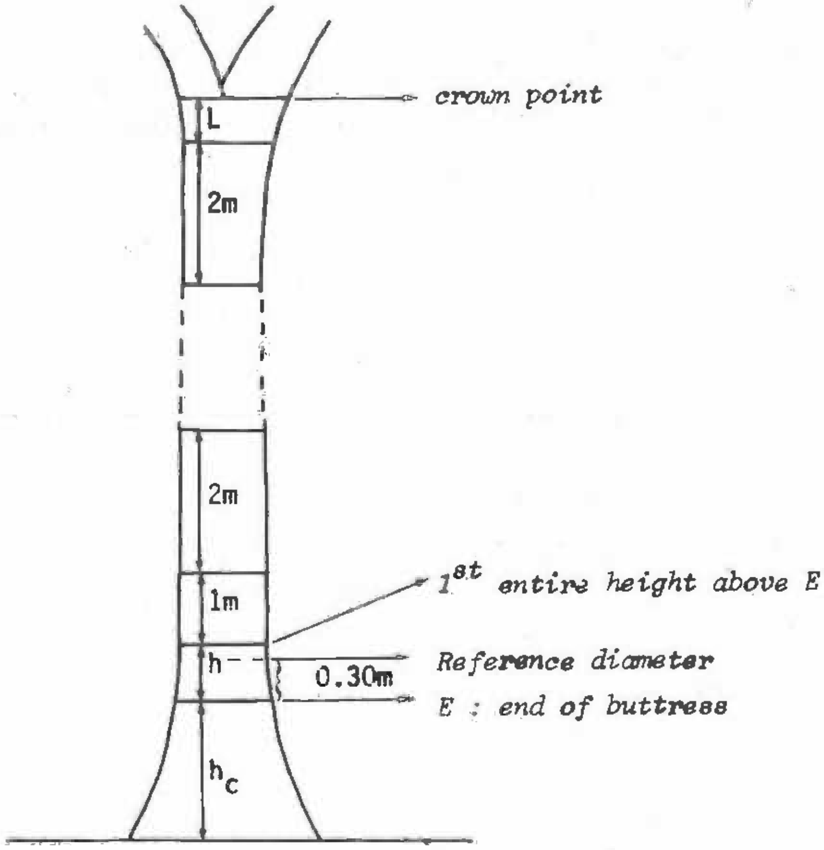
Don't measure the diameter if there is an anomaly.

5/ *Estimate the height* L *between last measurement and upper crosscut (base of the crown)* (L < 2m).

6/ *Measure diameter at upper crosscut.*

7/ *Indicate the parts of the bole which cannot be used (defect, insertion of a big branch...)*

8/ *Do the qualitative observations (see § 41)*



The measurements are gathered in the following form which is provided for the calculation of volume with a programming calculator : no place is therefore prepared for the volume of each log. Modify consequently the form if calculation is to be performed by hand.

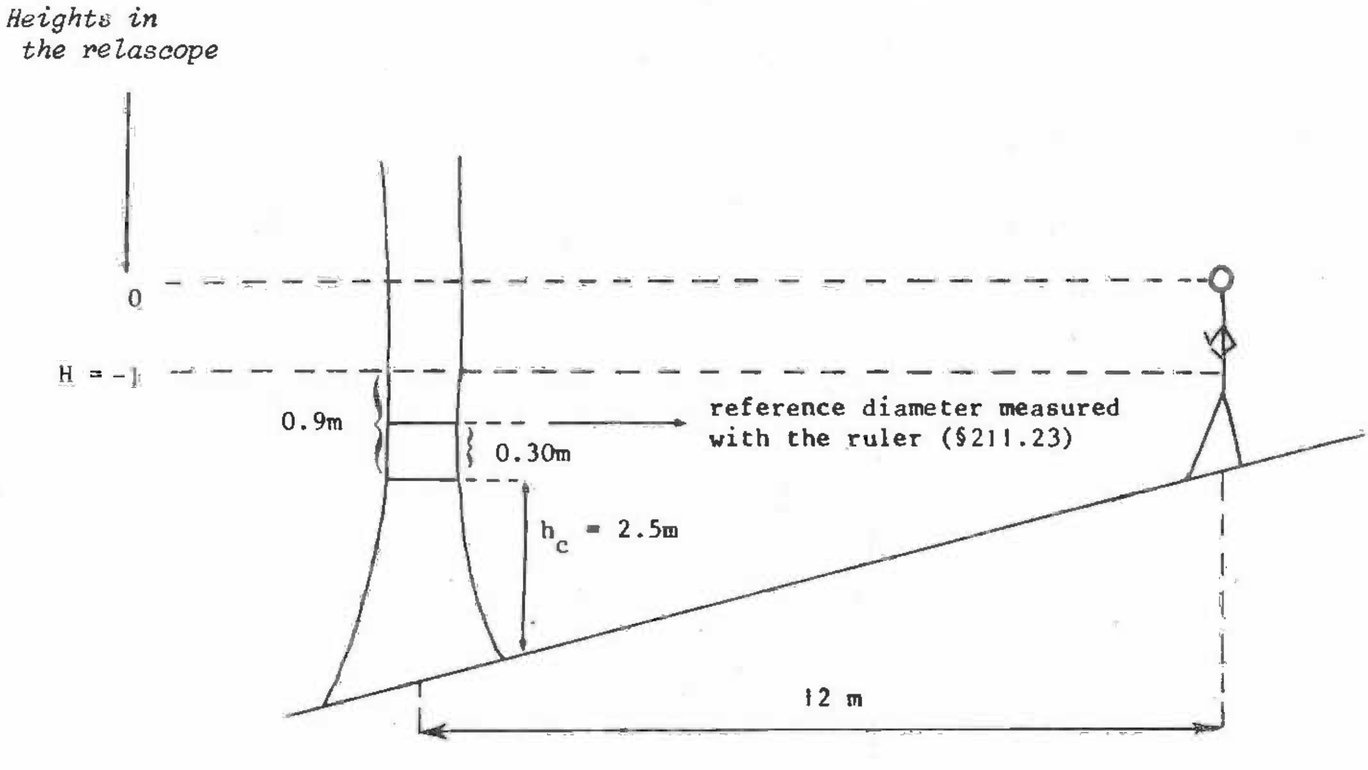
To transform the relascope-units into the real values, use the relationship :

one scale in cm = 2 × D in meters (ex. D = 10m, one scale = 20cm)

<u>Remark</u> :

For measurements by tree-climbing method, use a similar form. In the two columns for relascope-units, place overbark and underbark diameters.

<u>Example</u> : *The following form is for a tree which has been measured in the following conditions :*



Volume calculations have been made with Smalian's formula (see § 231) for each log limited by two measurements. For the bottom log (0.9 m high) the cylinder formula has been used. For the log with the broken branch, the length of the defective part has been estimated 1.8 m → length of good part = 2.2 m.

Bark thickness has been assumed constant from bottom to top.

Form for volume estimation of the bole of a standing tree with Bitterlich Relascope

Species | Code

Location : Block | Tran-sect | Plot

Mensurationist : | Date :

Horizontal distance : m

Qualitative observations: F H W

Bark
Height above ground m
Bark thickness on radius mm

Mark unusable parts

Relative heights

Relascope Units (R U): scale | 1/4 scale

2m
1m
1st entire height above E
H=
h m
E = end of buttress
h_c m

Relative heights

Relascope Units (R U): scale | 1/4 scale

R U: scale | 1/4 scale

reference diameter or | cm
reference circumference | cm
bole height | m
overbark volume | m3
underbark volume | m3
usable part: height | m
usable part: V overbark | m3
usable part: V underbark | m3

212 Height measurements

212.1 Definitions

The total height of a tree *is the length of the straight line connecting the foot of the tree (ground level) with the extremity of the terminal bud of the stem. For forked trees, there is a total height if the fork is above 1.30 m and as many heights as there are stems if the fork is below 1.30 m.*

In the same way as for volumes, the heights at certain crosscuts *are defined : the height "big wood" for example will be the length of the line connecting the foot of the tree with the 7 cm diameter crosscut of the stem.*

Remarks : . *for very badly formed trees or for shrubs with multiple stems as can be encountered in savannahs, the term diameter has little practical sense ; total height then becomes the essential characteristic.*

. *total height has little concrete sense for trees with a broken or dead crown. Avoid to use such trees to construct a volume table.*

212.2 Height measurements on standing trees

Height measurements take more time and are more delicate than diameter measurements. They are sometimes impossible (lack of visibility).

A height is measured :

- either using a system of graduated telescopic poles which is put against the tree. This is possible only for small heights (ranging about ten meters),
- or, most frequently, by optical procedure, using a dendrometer.

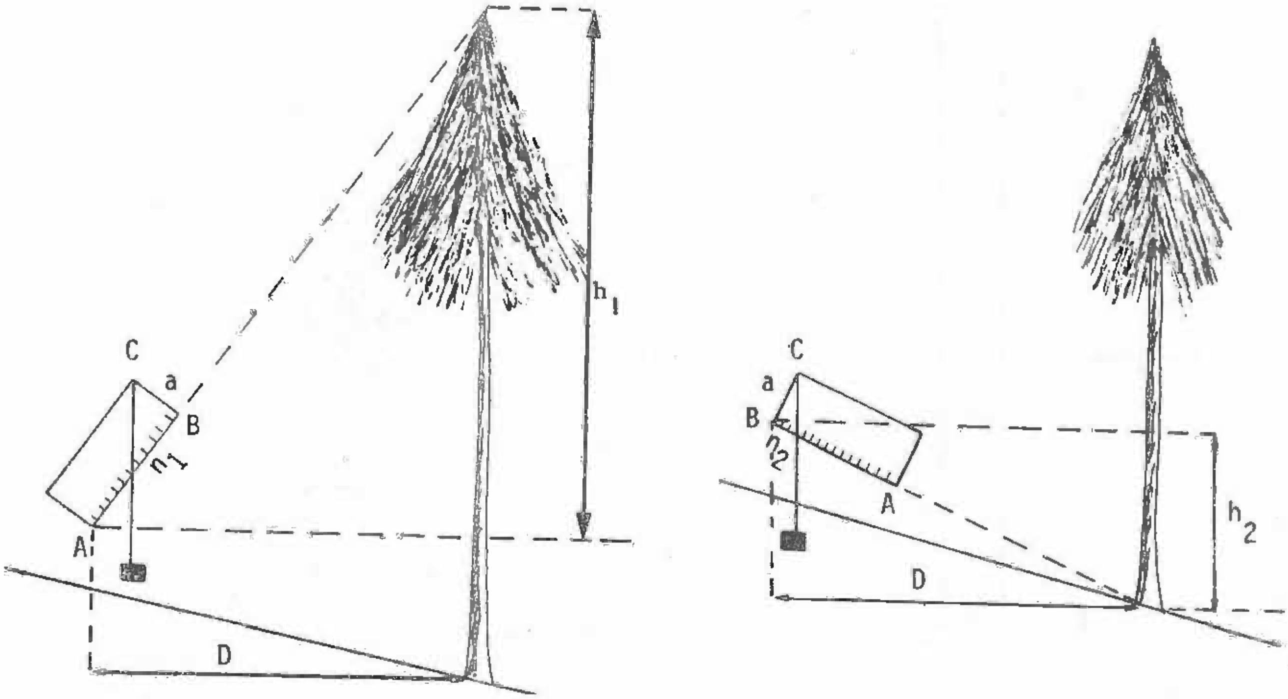
212.21 Some dendrometers

There exists a great variety of dendrometers. We will only describe the principle of two instruments easy to construct and quote some examples of commercial instruments.

212.211 Principle of two instruments easy to construct

- The dendrometric ruler

It is a ruler equiped with a plumb-line attached to one corner.



AB *is graduated in centimeters starting from* B *on both faces.*

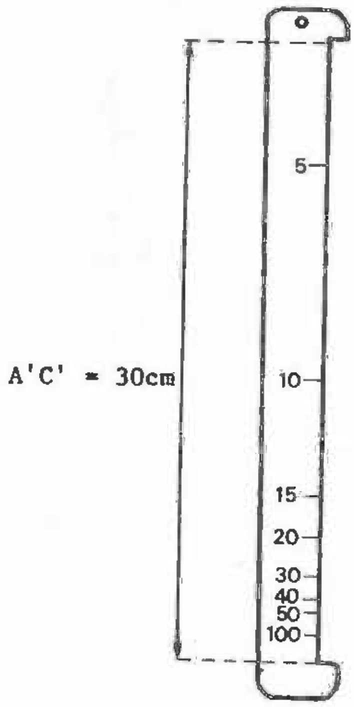
$H_{tot} = h_1 + h_2$ *with* : $h_1 = D \frac{n_1}{a}$ *and* $h_2 = D \frac{n_2}{a}$

If D = 10 m *and* a = 10 cm, H_{tot} *in meters is the sum* $n_1 + n_2$ *in centimeters.*

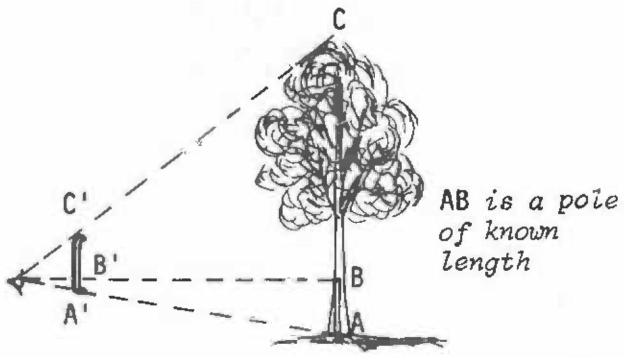
It is difficult to read the result with a precision better than half centimeter ; the error on the sum of the two measurements is therefore 1 cm maximum ; the error on the height is thus about 1 meter if D = 10 meters, *about 2 meters if* D = 20 meters , *etc...* *The use of this instrument is thus not recommended at more than 10 meters from the tree ; the maximum height which can be measured is then about 10 meters.*

This instrument requires the measurement of the distance from the tree. This is an instrument which avoids this measurement :

- The CHRISTEN hypsometer



graduations for a pole length AB of 4 meters



The observer stands at a distance such that the height to be measured is seen between A' *and* C'. *The instrument has to be held loosely so that it takes its vertical equilibrium position but should not move ; the height is read at* B' *on the scale.*

The longer A'C' , *the shorter the distance from the tree but it becomes more difficult to control simultaneously the* C'C , B'B *and* A'A *alignments.*

In general, the chosen length for the instrument is A'C' = 30 cm, *which leads one to stand at a distance from the tree approximately equal to the measured height.*

The scale is graduated according to the formula $A'B' = \frac{AB}{AC} A'C'$.

Here are some values of the A'B' *scale as a function of pole length* AB *and tree height* AC, *for an instrument of length* A'C' = 30cm.

AB → / AC ↓	3 m	4 m	5 m	6 m	7 m
5 m	180 mm	240 mm	300 mm		
6 m	150 mm	200 mm	250 mm	300 mm	
10 m	90 mm	120 mm	150 mm	180 mm	210 mm
11 m	82 mm	109 mm	136 mm	164 mm	191 mm
15 m	60 mm	80 mm	100 mm	120 mm	140 mm
16 m	56 mm	75 mm	94 mm	113 mm	131 mm
20 m	45 mm	60 mm	75 mm	90 mm	105 mm
21 m	43 mm	57 mm	71 mm	86 mm	100 mm
30 m	30 mm	40 mm	50 mm	60 mm	70 mm
31 m	29 mm	39 mm	48 mm	58 mm	68 mm
40 m	23 mm	30 mm	38 mm	45 mm	53 mm
41 m	22 mm	29 mm	37 mm	44 mm	51 mm

This table shows that the precision of the measurement decreases if the measured height increases and if pole length AB *decreases.*

In practice, this instrument is used only for heights lower than about 20 meters, which often is sufficient for measurements of bole heights in tropical high forest, because beyond that, the tree has little chance of being entirely visible and the distance from the observer to the tree becomes too large as can be seen with the following calculus : a precision better than x = 3 mm *on the* B' *reading seems hard to get. Let us impose that such an incertitude induces a* y *incertitude of no more than one meter on the* AC *result.*

The following relation between x *and* y *:*

$$AC = \sqrt{AB \times A'C' \times \frac{y}{x}}$$

shows that if A'C' = 30 cm *and* x = 3 mm, *the condition* $y \leq 1$ m *is satisfied if :*

AC ≤ 20 m for a pole AB of 4 meters

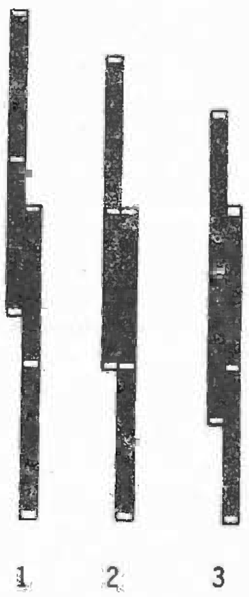
AC ≤ 22.4m for a pole AB of 5 meters

AC ≤ 24.5m for a pole AB of 6 meters.

212.212 Five commercial dendrometers

These instruments are, of course, more precise.

- The BLUME-LEISS dendrometer *is composed of a clinometer with pendulum which can be blocked at the moment of taking a sight in front of four scales graduated in heights and a fifth one in angles. The height scales correspond at a distance from the tree to be measured of 15, 20, 30 and 40 m. These distances can be measured with the aid of a diopter which gives 2 shifted images of a small, foldable target board which is hooked to the tree ; on this target are 3 lines* 45 cm apart *on one face and of 60 cm on the other, which corresponds, when the images of two lines come to coïncide, to distances of 15, 20, 30 or 40 meters.*



1 - too far

2 - correct distance

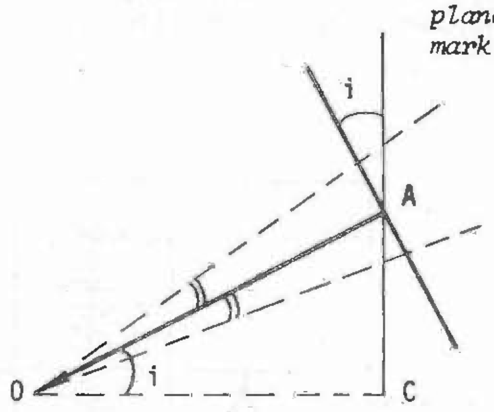
3 - too close

One takes a sight and when the pendulum has reached its position of equilibrum, it is immobilized by a knob. The height is given directly on the scale which corresponds to the distance.

On slopes, where the sight on the mark is inclined, the inclination is measured on the scale graduated in angles and the correction to be made to the height read is calculated according to a table which is engraved on the instrument. The mark is seen obliquely under an angle i *, the intercepted length of the mark is thus divided by* $\cos i$ *; on the other hand, a measure is taken of the oblique* OA *and not of the horizontal* $OC = OA \cos i$ *. The true height is therefore equal to the measured height multiplied by* $\cos^2 i$.

true height = measured height × $\cos^2 i$

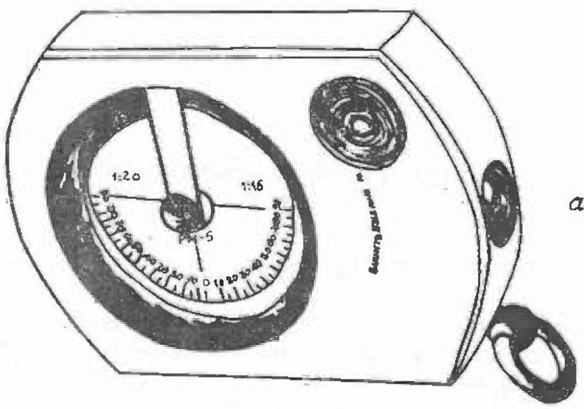
= measured height − measured height × $\sin^2 i$



A table carved on the instrument gives the values of $\sin^2 i$.

- The HAGA dendrometer *is practically identical but offers the advantage that only the scale corresponding to the chosen distance is visible, which eliminates the risk for error. A delicate point as regards these two instruments :* make sure that the action on the knob does not lock the pendulum in a position slightly different from its exact position.

- The SUUNTO dendrometer

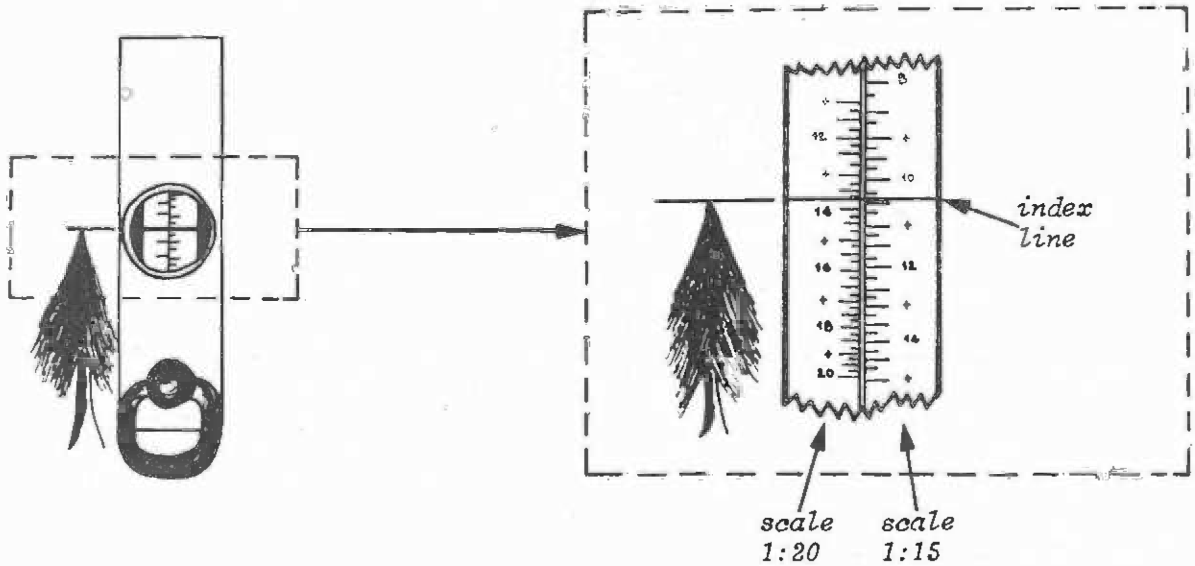


actual size

For establishing the measuring distance (15, 20, 30 or 40 meters) the instrument has a double-refracting prism and a separable calibrated target board made of reinforced plastic (the same that is used with the Blume-Leiss dendrometer). The target is fixed *vertically* on *the tree trunk at the eye level ; sight it through the prism and move backwards or forwards until the lines coïncide.*

Place the instrument to the eye and move it in a vertical arc until the horizontal index line, viewed through the lens, is aligned with the desired object.

Look simultaneously, with both eyes open, through the lens and alongside the instrument. The reading obtained is the height above the eye level.



Advantage over the Blume-Leiss dendrometer : *sighting and reading are simultaneous.*

Drawback : *sighting is more difficult.*

- *Any instrument which measures angles* ("cliseometer" or "clinometer") *can be used, the height in relation to the horizontal being the product of the horizontal distance to the tree by the tangent of the angle. Instruments where the reading is done on the moment of sighting are preferable to avoid the inconvenience mentioned caused by the knob. This is the case of the SUUNTO clinometer and the Bitterlich Relascope, instruments which are much used by foresters. The first one comprises a graduation of the angles in tangent ; one can stand at any distance from the tree but the product of the distance and the tangent has to be made. The relascope gives the height automatically if one stands at 20, 25 or 30 m from the tree (model with narrow scales) or at a distance equal to an even number of* *meters between 4 and 20 (model with wide scales).*

- *All these instruments give a maximum precision when one stands at a distance from the tree perceptibly equal to its height. This precision is, under optimum use conditions of each instrument, in the order of a few percent.*

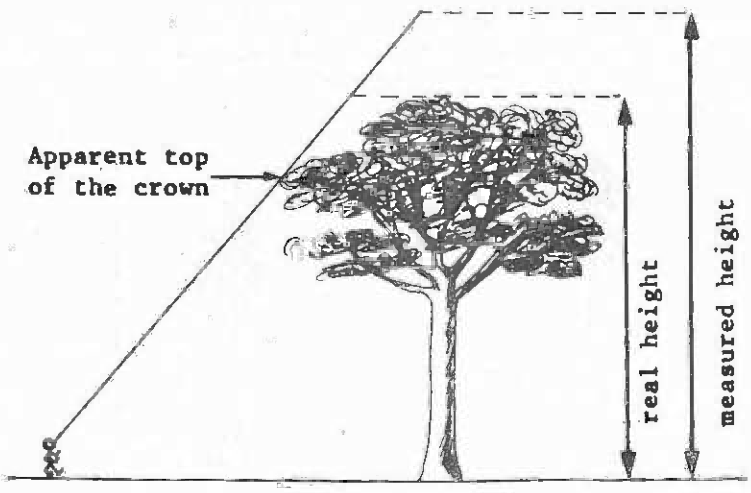
Recommendation : *calibrate each instrument as soon as it arrives ; it is not rare indeed to find out divergences as high as 3 % between two instruments of the same mark.*

212.22 Some practical remarks

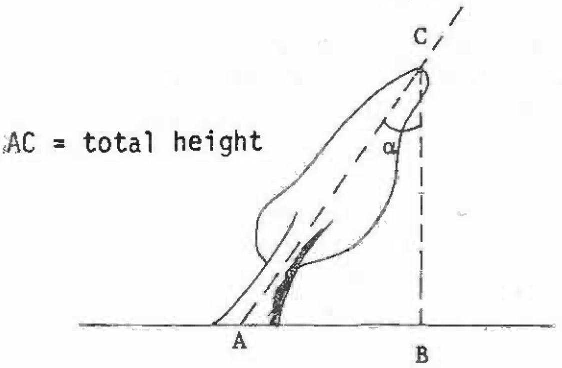
212.221 *Even if it is illusive to try to measure a total height with a precision better than the decimeter for small trees (a few meters) or than the meter for tall trees, it is advisable, in order to lose the least possible precision, to do the measurement with the maximum precision permited by the instrument used, let us say tentatively :*

- to the nearest cm for trees of less than 2 meters high,
- to the nearest dm for trees of height between 2 and 5 m,
- to the nearest $\frac{1}{2}$ m for trees of height between 5 and 10 m,
- to the nearest meter for trees higher than 10 m.

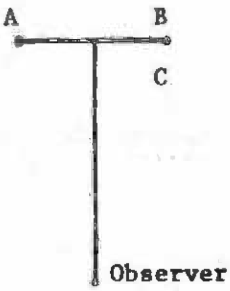
212.222 *Measure a total height only if the top of the crown can be seen ; if an apparent top is viewed, the measured height overestimates the real height. This overestimate can be very important ; it is about 20 % in this case.*



212.223 *Height measurement on a leaning tree.*



From above :



The observer should not be in the vertical plane defined by the tree but perpendicularly to this plane, at equal distance from A *and* B.

With the CHRISTEN dendrometer (or any other instrument which does not necessarily measure vertical distances), the reference pole is put alongside AC *and the observer takes a sight on* AC, *which gives the exact height.*

With a dendrometer which measures vertical distances only (Blume-Leiss, Bitterlich relascope, SUUNTO dendrometer,...) a correction is theoretically necessary because the measured height is BC *and not* AC *:*

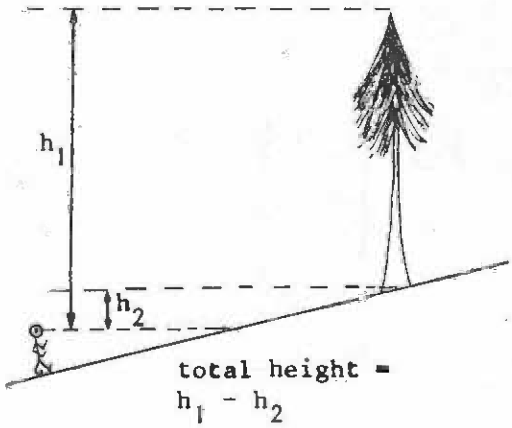
$$AC = \sqrt{AB^2 + BC^2} = \frac{BC}{\cos \alpha}$$

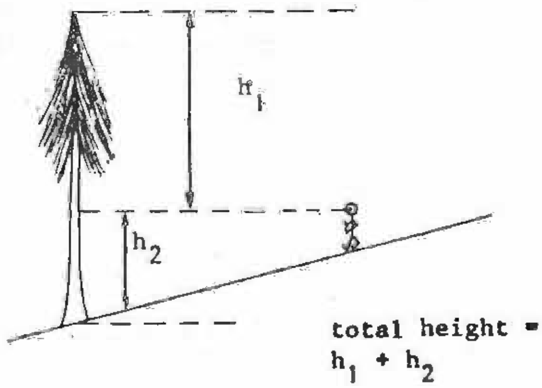
but this correction is generally small :

$$\text{Relative error} = \frac{AC - BC}{AC} \approx 1 - \cos \alpha \begin{cases} = 1.5\ \% \text{ for } \alpha = 10° \\ = 3.4\ \% \text{ for } \alpha = 15° \end{cases}$$

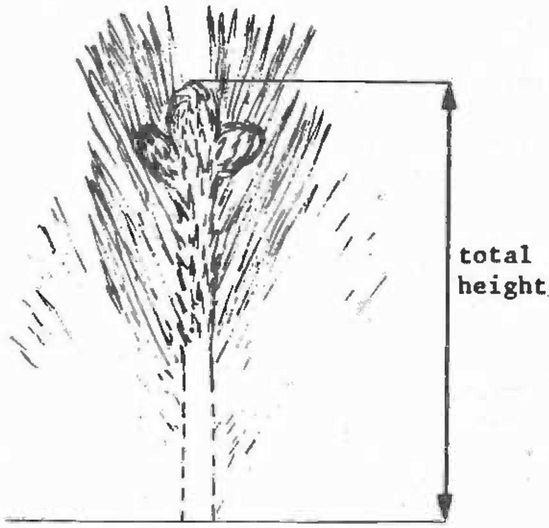
212.224 *Dendrometers which measure height above observer's eye.*

From or to the measurement of the top, it is necessary to substract or add the measurement of the base depending on whether the eye is beneath or above the foot of the tree :





On horizontal terrain, the measurement of the foot of the tree need not be done because h_2 *is then known (distance from the eye to the ground).*



Example of a young pine

212.225 *The definition of total height involves the extremity of the terminal bud of the stem ; this is not necessarily the highest point of the tree. The distinction has a practical consequence (which can be important) only for the small trees the tip of which can be reached by hand.*

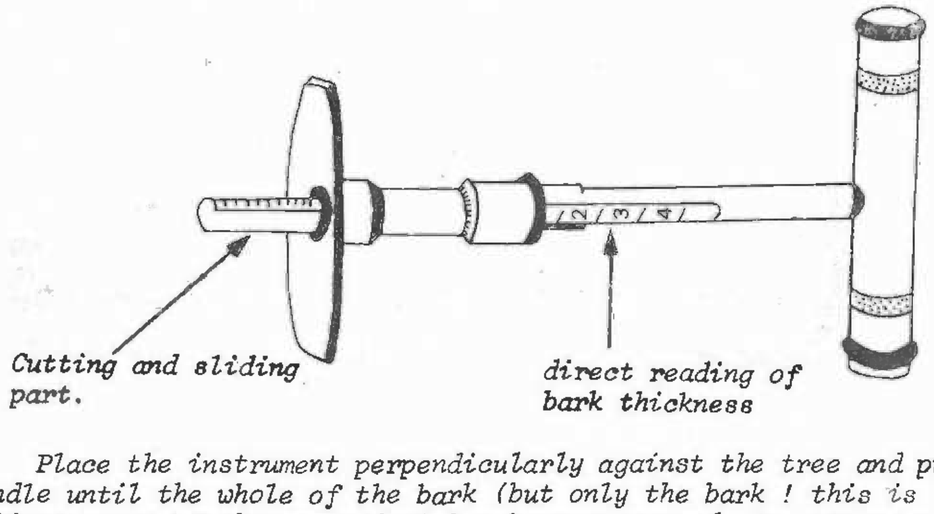
213 Measurement of bark thickness

To know the volume under bark is a necessity if it is the utilizable volume that one wants to know because the bark is generally not utilized.

The proportion of bark volume over the volume with bark varies from a few percent to approximately twenty percent for the majority of species. This proportion is all the more important if the tree is young, if the altitude increases and, in a general way, if the growing conditions are more difficult.

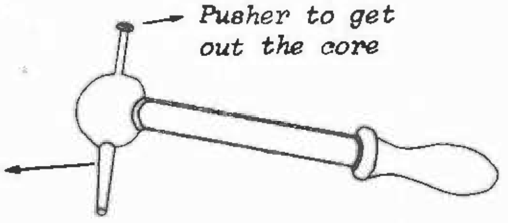
Instruments have been specially designed to measure the bark thickness. They measure thickness on the radius (maximum capacity approximately 5 cm) but be careful : some instruments are graduated to give double bark thickness.

a/ The bark gauge



Place the instrument perpendicularly against the tree and push the handle until the whole of the bark (but only the bark ! this is the delicate part of the operation) has been traversed. Do not use a *mallet to make the job easier. It is better to do two measurements, in two points diametrically opposed, and to take the arithmetic mean.*

b/ *This is an other instrument, the* "boring hammer", *because it is used as a hammer, the cutting tube having to hit the tree at a right angle.*



This instrument has been conceived to take rapidly small wood cores but it is sometimes used for rapid measurements of thin barks (approximately 2 cm thickness maximum). This is not to be recommended because measurements can be very inaccurate.

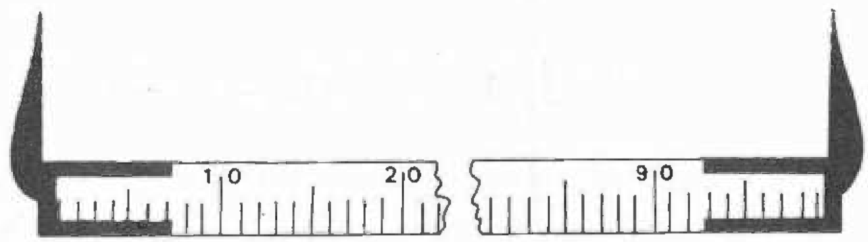
22 MEASUREMENTS ON FELLED TREES

Important remark : Whatever the measurements may be that are carried out on a felled tree, its reference diameter has to be known.

If possible, the reference diameter has to be measured before felling,
if not, reconstitute, examining the stump, which was the height of the reference diameter and take the measurement there.

221 Length measurements

Length measurements are carried out with a decameter tape and are given in meters with at least one decimal place (round off to the nearest dm, or to the nearest cm) ; sometimes also a graduated ruler is used of one meter long, equipped with a steel pin on each edge : streching alternatively each pin, a single person can rapidly take the measurement.



222 Size measurements

Size measurements are also carried out with a tape, good care being taken that the tape is put perpendicularly on the axis of the stem and closely fitted on the whole of the periphery. If it is difficult to slide the taper under the tree, even when using a curved steel needle attached to the tape, the diameter is measured with a caliper.

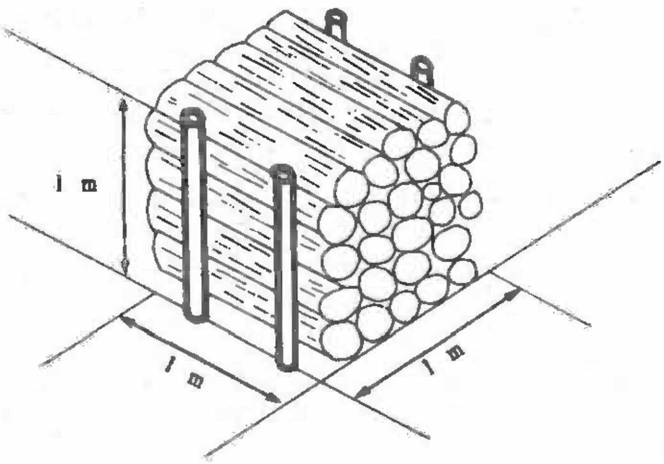
223 Accurate measurement of the diameters under bark or under sapwood

Using a bark measurement instrument is of course possible, but one can take advantage of the fact that the tree is felled to debark the tree or to cut of the sapwood and to take measurement of the diameters under bark and/or under sapwood at various heights.

224 Mensuration of stacked wood

The volume obtained is expressed in "stacked cubic meters" *with one decimal place.*

A "stacked cubic meter" *is the bulk volume occupied by pieces of wood one meter long piled on one meter width, one meter high.*



It is thus a volume which contains air and wood in variable proportions according to the form of the pieces. The piling coefficient *is the volume of wood expressed in m3 contained in a* "stacked cubic meter" *if all pieces were cylindrical and of the same diameter, the piling coefficient would be :* $\frac{\pi}{4} = 0.785$ *. In practice it varies between* 0.45 *(small branches of bad form) and* 0.80 *(split cordwood piled small end to large end).*

It is difficult to estimate precisely a piling coefficient. Here are some indications to estimate the volume of wood in a parallelepipedic pile. For further details, see bibliography, references 9 and 13.

- If the pieces are not too small, *take on each of them the following measurements :*
  - *diameter at each end and in the middle*
  - *and apply Newton's formula (see § 231).*

This is tedious and forces to pull down the pile. More simply, measure diameters of every piece on both faces of the pile (don't try to associate the two measurements of a piece) : Smalian's formula (see § 231) gives :

$$\text{Volume of wood in the pile} = \frac{\pi}{8} \underset{\substack{\downarrow \\ \textit{length of pieces}}}{L} \left\{ (\Sigma D^2)_{\text{face 1}} + (\Sigma D^2)_{\text{face 2}} \right\}$$

Carry out this operation on some similar piles and take for piling coefficient :

$$p = \frac{\text{Total volume of wood contained in the piles}}{\text{Sum of the volumes of the piles in "staked cubic meters"}}$$

A confidence interval for p *can be estimated. See a manual on sampling techniques, chapter* "ratio estimates". *For instance : ref. 7.*

It is possible also to weigh the piles (the wood is sometimes sold by weight), which allows density estimation.

- If the pieces have small diameters, *weighting only is practicable but the problem of volume estimation remains if the piled wood must be expressed in cubic meters : Immerse the wood and measure the volume of water displaced...*

Remark : *when stacked wood is cut by axe leaving* "pencil point" *ends, take for length of the piece the length without the points.*

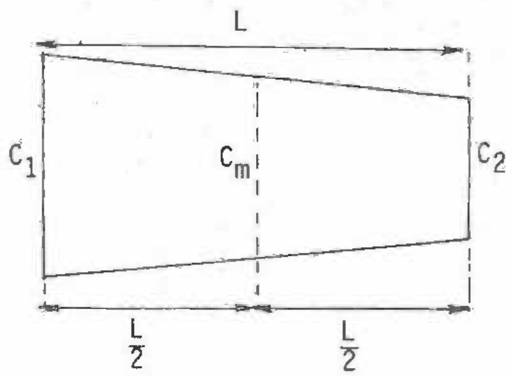
Attention : *the shorter straighter or fatter the piled pieces, the higher the piling coefficient. If for example a pile consists of pieces of wood 2 m long, a coefficient which has been calculated for the same pieces but of only 1 m long cannot be used ; the difference between the two piling coefficients is often important (in the order of 20 percent more for pieces of 1 m than for pieces of 2 m, but this has to be verified in each case).*

23 DIRECT CALCULATION OF THE VOLUME OF A TREE FROM MEASUREMENTS TAKEN ON THE TREE

231 Calculating procedures

The volume sought will be obtained by adding volumes of its components. The basic calculation therefore consists in calculating the volume of a log (stem or branch).

Volume of a log of length L



C_1 and C_2 are the girths at the extremities.

C_m is the girth at mid-length

D_1, D_2, D_m are the corresponding diameters.

Various calculation methods are possible :

If C_m is known
$$V = \frac{C_m^2}{4\pi} L = \frac{\pi}{4} D_m^2 L \qquad (1) \text{ Huber}$$

If C_1 and C_2 are known
$$V = \frac{1}{4\pi} \left(\frac{C_1^2 + C_2^2}{2}\right) L = \frac{\pi}{4} \left(\frac{D_1^2 + D_2^2}{2}\right) L \qquad (2) \text{ Smalian}$$

$$V = \frac{1}{4\pi} \left(\frac{C_1 + C_2}{2}\right)^2 L = \frac{\pi}{4} \left(\frac{D_1 + D_2}{2}\right)^2 L \qquad (3)$$

$$V = \frac{1}{12\pi} (C_1^2 + C_2^2 + C_1C_2) L = \frac{\pi}{12} (D_1^2 + D_2^2 + D_1D_2) L \qquad (4) \text{ formula for a truncated cone}$$

If C_1, C_2 and C_m are known
$$V = \frac{L}{24\pi} \left(C_1^2 + 4C_m^2 + C_2^2\right) = \pi \frac{L}{24} \left(D_1^2 + 4D_m^2 + D_2^2\right) \qquad (5) \text{ Newton - Simpson}$$

Remark that (3) < (4) < (2) *and that* $(4) - (3) = \frac{(2) - (4)}{2} = \frac{\pi L}{48} (D_1 - D_2)^2$

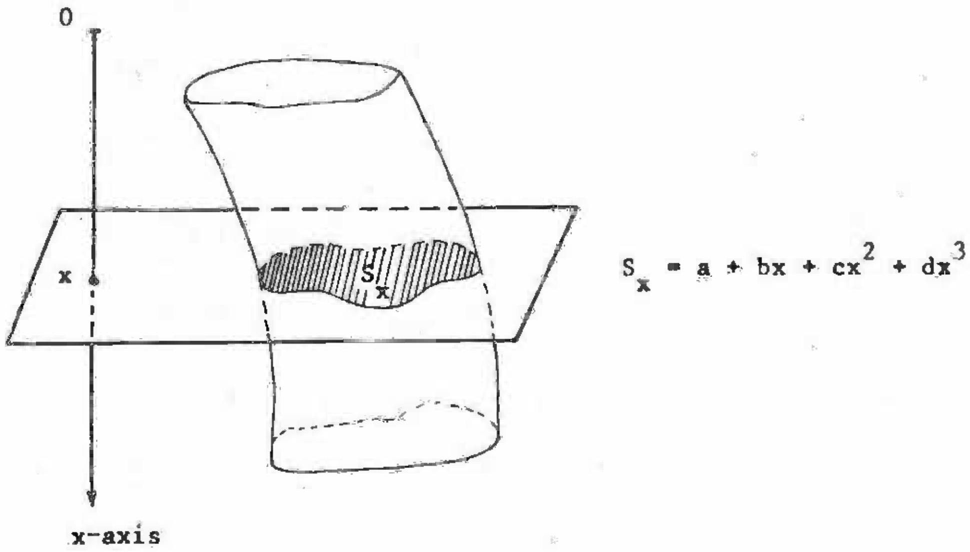
Let us see what these formulas give in some classical cases :

If the log is :	*formula (1)*	*formula (2)*	*formula (3)*	*formula (4)*	*formula (5)*
A cylinder $C_1 = C_m = C_2$	is exact	is exact	is exact	is exact	**is exact**
A parabolic log $C_m^2 = \frac{1}{2}\left(C_1^2 + C_2^2\right)$	is exact	is exact	underestimates the real volume with 3ε (*)	underestimates the real volume with 2ε (*)	**is exact**
A conic log $C_m = \frac{1}{2}\left(C_1 + C_2\right)$	underestimates the real volume with ε (*)	overestimates the real volume with 2ε (*)	underestimates the real volume with ε (*)	is exact	**is exact**
A neloidic log $C_m^{2/3} = \frac{1}{2}\left(C_1^{2/3} + C_2^{2/3}\right)$	(*) underestimates the real volume with $\frac{3\varepsilon - \varepsilon'}{2} > \frac{4\varepsilon}{3} > 4\varepsilon'$	(*) overestimates the real volume with $3\varepsilon - \varepsilon' > \frac{8\varepsilon}{3} > 8\varepsilon'$	(*) underestimates the real volume with $\varepsilon' < \frac{\varepsilon}{3}$	(*) overestimates the real volume with $\varepsilon - \varepsilon' > \frac{2\varepsilon}{3} > 2\varepsilon'$	**is exact**

with : $\varepsilon = \frac{\pi L}{48} (D_1 - D_2)^2$ *and* $\varepsilon' = \frac{\pi L}{16} \left\{ D_1^{2/3} D_2^{1/3} - D_1^{1/3} D_2^{2/3} \right\}^2$

$(\varepsilon > 3\varepsilon')$

Formula (5) is exact for each solid (not necessarily of revolution) for which the area of the section is a cubic of the distance of this section to a section origin.



This is the case for the cylinder, the paraboloid, the cone and the neloid, because these solids are obtained by rotation around the x-axis of a curve $y = ax^b$ *with :*

cylinder : $b = 0$ *which gives* $S_x = \pi a^2$

paraboloid : $b = \frac{1}{2}$ *which gives* $S_x = \pi a^2 x$

cone : $b = 1$ *which gives* $S_x = \pi a^2 x^2$

neloid : $b = \frac{3}{2}$ *which gives* $S_x = \pi a^2 x^3$

One would thus think that, to calculate the volume of a stem of which the girths at a spacing of length L *are known, it would be preferable to apply formula (5) for logs of length* 2 L *instead of one of the formulas (1) to (4) for logs of length* L . *This is not necessarily so since the conditions of validity of Newton's formula, though rather general, are not necessarily fulfilled by each log.*

As a matter of fact, formula (5) being somewhat less easy to use, one rather uses the others. Which one is the best ? One cannot answer this question which, moreover, is of small importance because the precision of the estimation of the volume depends more from the diameter measurements (precision and number) than from the calculating method used.

Remark : *The 5 formulas can be considered to give similar results if* $\frac{C_2}{C_1} > 0.82$ *because for each case marked * in the previous table, the relative error is then less than 1 %.*

Whichever way a volume has been calculated, it should be expressed in cubic meters, with 3 or 4 decimal places.

232 Recommendations for the measurements to be taken with regard to the required volumes

The following recommendations are made about measurements to be taken on a tree for a direct estimation of its volume.

	Required volume (with bark)		
	Total stem volume	Stem volume at a fixed cross cut (for example : big wood volume)	Total and big wood stem + branches volume
STANDING TREE (optical measurements)	[1] Compulsory: . Reference diameter D_R . Stump diameter D_S . Length between D_R and D_S . A diameter at a higher level than D_R, for example, diameter at about 1/4 H_{tot} or 1/2 H_{tot} (preferable). . H_{tot} = total height. If possible: Diameter at other heights ; as provided for on the form for the mensuration of standing trees with the Bitterlich relascope (§ 211.25) : diameter every 2 m with an intercalated measurement in the lower part.	[2] Compulsory: . Reference diameter D_R . Stump diameter D_S . Length between D_R and D_S . H_{cr} = height at cross cut . A diameter at a higher level than D_R, for example diameter at about 1/4 H_{cr} or 1/2 H_{cr} (preferable). If possible: Diameter at the cross cut and diameters at other heights.	This volume can only be measured on a felled tree.
FELLED TREE	[3] Compulsory: . Reference diameter D_R . Stump diameter D_S . Length between D_R and D_S . H_{tot} = total height . Diameter at 1/2 H_{tot} If possible: Diameter every meter or every two meters from the stump	[4] Compulsory: . Reference diameter D_R . Stump diameter D_S . Length between D_R and D_S . H_{cr} = height at cross cut . Diameter at H_{cr} and 1/2 H_{cr}. If possible: Diameter every meter or every two meters from the stump	Stem : see [3] and [4] Branches : two possibilities - proceed for each large branch as for the stem and stack the small branches - more simply : stack all the branches

Whatever the volume required, it is good to take always the measurements enumerated under [1] *; they allow, by interpolation, the calculation of the stem volume comprised between any lower and any upper cross cut.*

Remarks :

1 - *The volume under bark can only be obtained accurately on a felled tree because the bark thickness can be measured at any height. On a standing tree the bark is measured at reference height and some assumption is made on the decrease of bark thickness taking into account data collected on felled trees. Without any such information an assumption has to be made (for instance = constant bark thickness). See paragraph 36.*

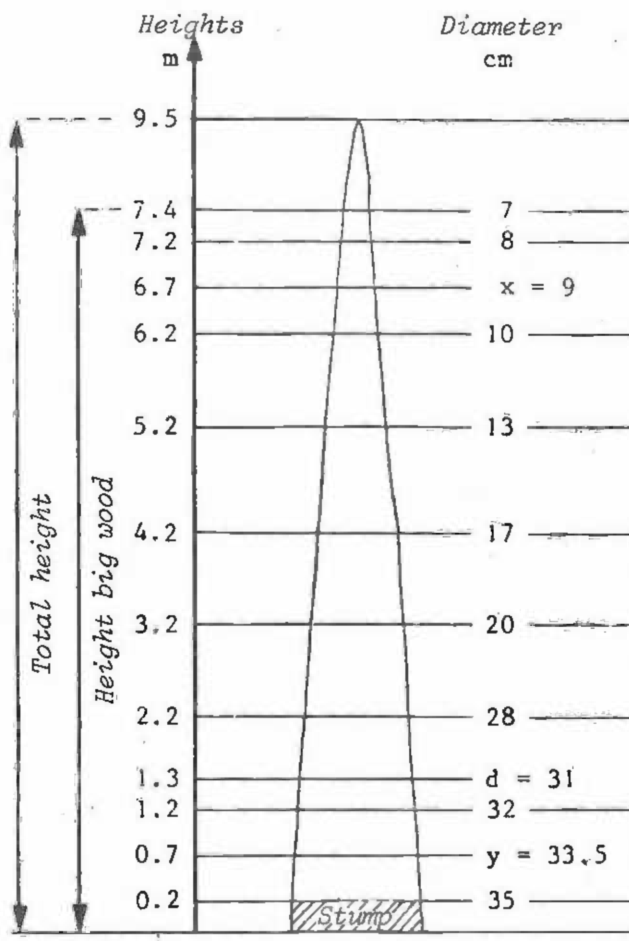
2 - *The measurements given in the table allow calculation of gross volumes. To get net volumes, the additional measurements to be taken depend on the type of net volume required. The form given in paragraph 211.25 for volume calculation on standing trees with Bitterlich relascope shows a simple way to record the defective parts of the bole of a high-forest tree.*

3 - *Problem raised by the stump. The felling level depends on species, tree size, local habits and changes with sawing equipment. Thus, a volume calculated on a standing tree contains an uncertainty due to the unknown stump height. In order to be able to calculate easily the volume in function of different hypothesis on stump height, a simple way (for trees without buttress only) is to give a volume in which the part under reference diameter* D_R *is the cylinder between* D_R *and the ground and with diameter* D_R*. From this volume which contains the stump, it is easy, knowing the felling level, to derive the felled volume. See example 233.2.*

233 Examples

233.1 *Let us take a felled tree on which have been measured overbark diameters every meter starting from the stump. Let us calculate the total volume and the "big wood" volume of the stem.*

Here are 5 different calculating methods for the tree divided into 1 m logs.



Volume in m³ | [n] : Formula used

Heights m	Method 1	Method 2	Method 3	Method 4
9.5 – 7.4	[4] 0.0027	[4] 0.0027	[4] 0.0027	[4] 0.0027
7.4 – 7.2	[2] 0.0009	[3] 0.0009	[2] 0.0009	[2] 0.0009
7.2 – 6.2	[2] 0.0064	[3] 0.0064	[2] 0.0064	[5] 0.0166
6.2 – 5.2	[2] 0.0106	[3] 0.0104	[5] 0.0279	
5.2 – 4.2	[2] 0.0180	[3] 0.0177		[5] 0.0452
4.2 – 3.2	[2] 0.0271	[3] 0.0269	[5] 0.0700	
3.2 – 2.2	[2] 0.0465	[3] 0.0452		[5] 0.1194
2.2 – 1.2	[2] 0.0710	[3] 0.0707	[5] 0.1598	
1.2 – 0.2	[2] 0.0883	[3] 0.0881		[2] 0.0883
Total stem volume m3	0.2715	0.2690	0.2677	0.2731
Big wood stem volume m3	0.2688	0.2663	0.2650	0.2704

heights m	Method 5
9.5 – 7.4	[4] 0.0027
7.4 – 7.2	[2] 0.0009
7.2 – 6.7	[2] 0.0028
6.7 – 5.7	[1] 0.0079
5.7 – 4.7	[1] 0.0133
4.7 – 3.7	[1] 0.0227
3.7 – 2.7	[1] 0.0314
2.7 – 1.7	[1] 0.0616
1.7 – 0.7	[1] 0.0804
0.7 – 0.2	[2] 0.0461
Total stem volume m3	0.2698
Big wood stem volume m3	0.2671

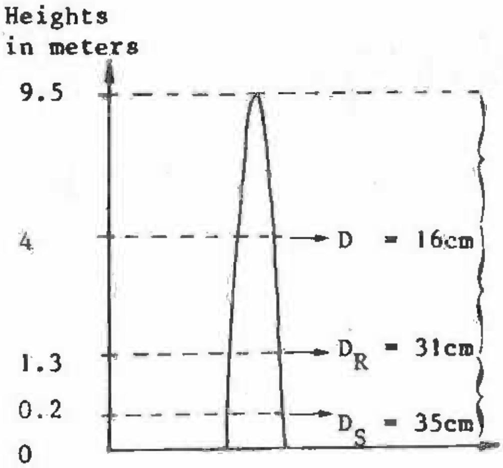
Note : d = *reference diameter = 31 cm*

x, y = *estimated diameter (by average) used in method 5*

The 5 methods give very similar results (this is always the case when the logs are short). Of course, it cannot be said which one is the nearest to the truth.

233.2 *Measurements on a standing tree for total stem volume. Case where only compulsory measurements are available (cell* [1] *of table of paragraph 232).*

For the upper log, an hypothesis is needed on the form. Let us suppose a conic form.

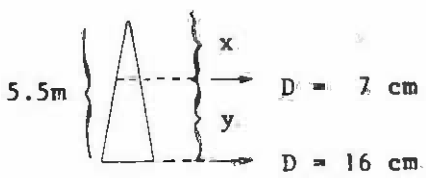


Cone formula	: $\frac{\pi}{12} \times 0.16^2 \times 5.5$	= 0.03686
Smalian's formula	: $\frac{\pi}{4} \left(\frac{0.16^2 + 0.31^2}{2}\right) 2.7$	= 0.12904
Cylinder with diameter 31cm	: $\frac{\pi}{4} \times 0.31^2 \times 1.3$	= 0.09812
		0.2640

$\Longrightarrow$ total stem volume = $\left(0.2640 - \frac{\pi}{4} \times 0.31^2 \times H_S\right)$ m3

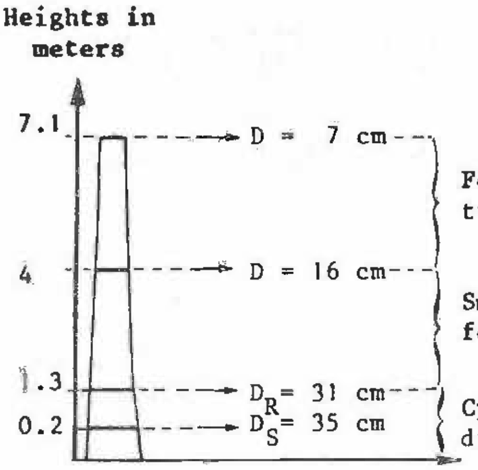
H_S ↓ *stump height in meters*

To estimate the big wood volume of the stem without knowing the height of crosscut D = 7 cm *, this height has first to be estimated, which necessitates an hypothesis on the form of the last log ; let us suppose again the conic form :*



$\frac{x}{5.5} = \frac{7}{16} \longrightarrow$ 2.4 m and y = 3.1 m

Calculation of big wood volume of the stem :



Formula for a truncated cone : $\frac{\pi}{12}\left(0.07^2+0.16^2+0.07\times0.16\right)3.1 = 0.03384$

Smalian's formula : $\frac{\pi}{4}\left(\frac{0.16^2 + 0.31^2}{2}\right) 2.7 = 0.12904$

Cylinder with diameter 31cm : $\frac{\pi}{4} \times 0.31^2 \times 1.3 = 0.09812$

0.26100

$\Longrightarrow$ Big wood volume of the stem : $(0.2610 - \frac{\pi}{4} \times 0.31^2 \times H_S)$ m3

↑ stump height in meters

In these two volume calculations, the D_S *measurement is not used as the tree below* D_R *is assumed to be a cylinder. However, the data* D_R , D_S , length between D_R and D_S , *are useful to study the form of the stem basis.*

24 STUDY OF TREE FORM

Reference diameter and total height cannot suffice to describe completely the form of a tree. We shall restrict ourselves to the study of the form of the stem because it is impossible to recommend for every species a single method to describe the morphology of every constitutive part of a tree ; moreover, such observations (for instance : number, insertion angle and straightness of branches, etc...) are mainly useful for geneticists and this is out of the scope of this manual.

241 Measure of stem form with a coefficient

241.1 Definitions

The simplest coefficient is the form factor f :

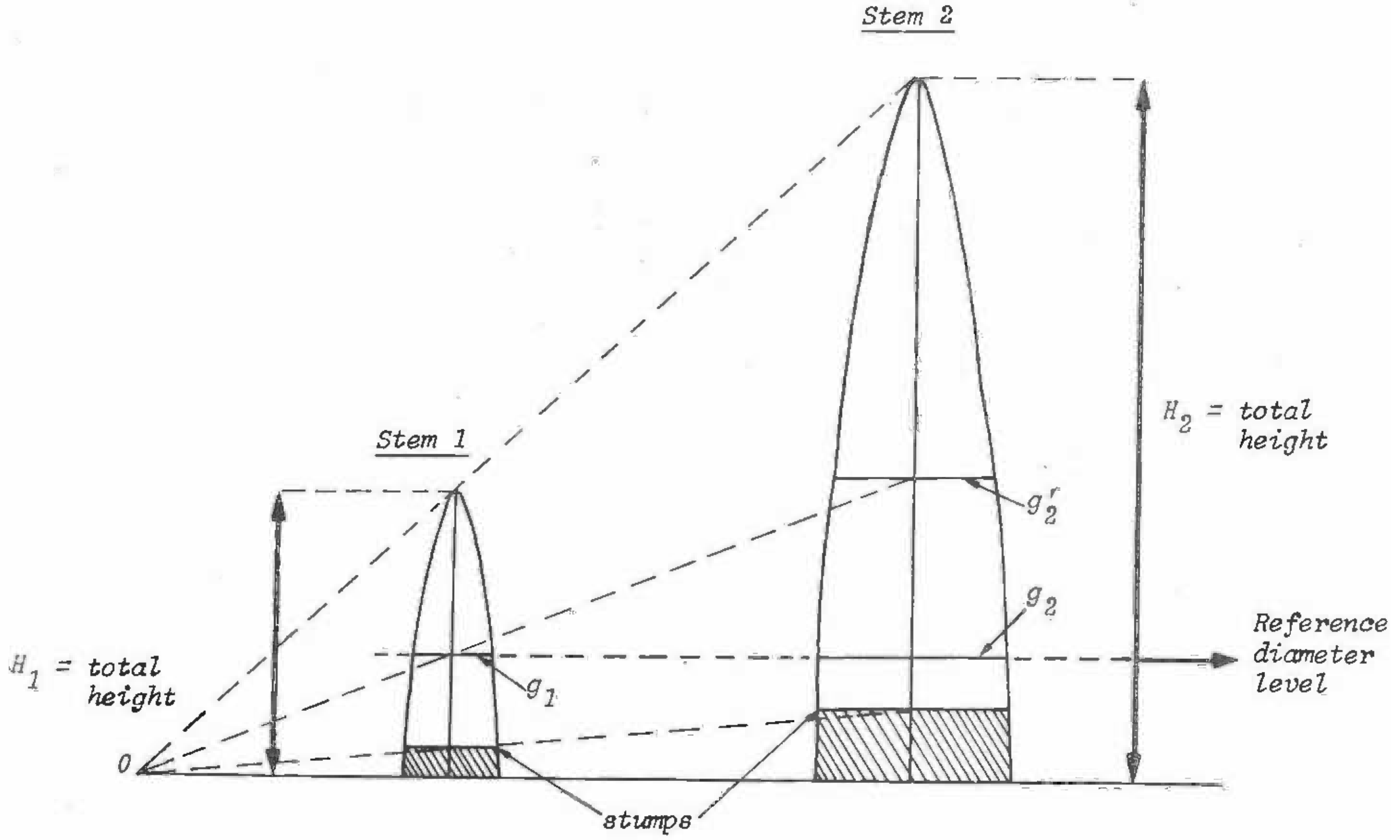
$$f = \frac{\text{Volume}}{\text{(Reference basal area)} \times \text{total height}}$$

Thus, every volume which can be defined in a tree has a corresponding form factor. The most common is related to the total volume of the stem but the form factor corresponding to the volume of the stem until a given upper limit may however be considered.

The form factor f *is not a characteristic of stem form :*

(a) : *two stems with same* f *do not have necessarily the same form and mainly :*

(b) : *two stems of same form do not have the same* f *; let us indeed consider two stems of same form (similar stems).*



The similitude with 0 as center and $k = \frac{H_2}{H_1}$ *as ratio transforms stem 1 into stem 2.*

The stem-volumes are V_1 *and* V_2 *(stump volume is included or not ; this does not affect the result) and the corresponding form factors are :*

$$f_1 = \frac{V_1}{g_1 H_1} \quad \text{and} \quad f_2 = \frac{V_2}{g_2 H_2}$$

The relationships: $V_2 = k^3 V_1$, $H_2 = k H_1$ and $g_2' = k^2 g_1$

show that : $\frac{f_1}{f_2} = \frac{g_2}{g_2'}$

Since $g_2 > g_2'$, f_2 *is less than* f_1.

Hohenadl removed drawback (b) *by defining a coefficient, called* stem-natural form factor *(as opposed to* f *which is sometimes called artificial form factor) :*

$$f' = \frac{\text{stem volume}}{g_{\frac{H}{10}} \times H}$$

where : H = total height

$g_{\frac{H}{10}}$ = area of stem section at height $\frac{H}{10}$

However, the difficulty of measuring a diameter at a relative height on a standing tree limits the use of the f' *coefficient.*

Remark : *the natural form factor corresponding to the total volume of a stem is generally between 0.3 and 0.6. It is equal to :*

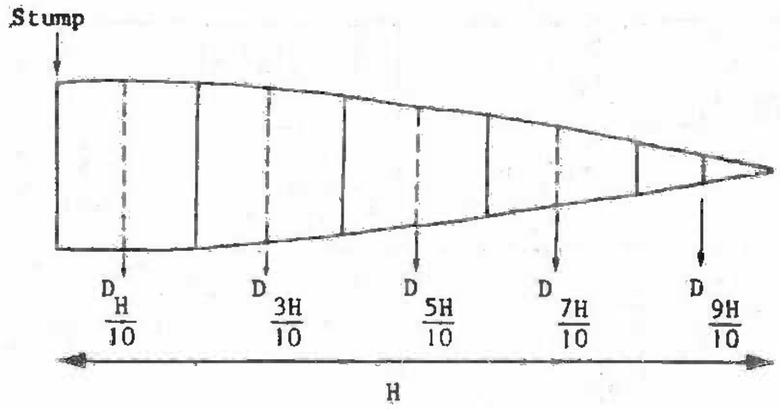
$\frac{5}{9}$ *= 0.56 for a paraboloïd*

$\frac{100}{243}$ *= 0.41 for a cone*

$\frac{250}{569}$ *= 0.34 for a neloïd.*

241.2 How to calculate f or f'

The form factor of a stem cannot be measured directly ; the volume has to be calculated first. It has been said in paragraph 23 that the principle is to divide the stem into logs and to add the volumes of these logs. For instance, if the stem is divided into 5 logs of equal length and if the diameter is measured at the middle of each :



the application of Huber-formula to each log gives :

$$f = \frac{1}{5}\,\frac{D^2_{\frac{H}{10}} + D^2_{\frac{3H}{10}} + D^2_{\frac{5H}{10}} + D^2_{\frac{7H}{10}} + D^2_{\frac{9H}{10}}}{(\text{reference diameter})^2}$$

$$f' = \frac{1}{5}\,\frac{D^2_{\frac{H}{10}} + D^2_{\frac{3H}{10}} + D^2_{\frac{5H}{10}} + D^2_{\frac{7H}{10}} + D^2_{\frac{9H}{10}}}{D^2_{\frac{H}{10}}}$$

In order to calculate the average form factor of a set of n trees, several calculus are possible. Here are three of them :

	Mean of natural form factor	Mean of artificial form factor
(1)	$\dfrac{\sum_{i=1}^{n} f'_i \left(D^2_{\frac{H}{10}} H\right)^2_i}{\sum_{i=1}^{n} \left(D^2_{\frac{H}{10}} H\right)^2_i} = \dfrac{\sum_{i=1}^{n} V_i \left(D^2_{\frac{H}{10}} H\right)_i}{\frac{\pi}{4} \sum_{i=1}^{n} \left(D^2_{\frac{H}{10}} H\right)^2_i}$	$\dfrac{\sum_{i=1}^{n} f_i \left(D^2_R H\right)^2_i}{\sum_{i=1}^{n} \left(D^2_R H\right)^2_i} = \dfrac{\sum_{i=1}^{n} V_i \left(D^2_R H\right)_i}{\frac{\pi}{4} \sum_{i=1}^{n} \left(D_R H\right)^2_i}$
(2)	$\dfrac{\sum_{i=1}^{n} f'_i \cdot \left(D^2_{\frac{H}{10}} H\right)_i}{\sum_{i=1}^{n} \left(D^2_{\frac{H}{10}} H\right)_i} = \dfrac{\sum_{i=1}^{n} V_i}{\frac{\pi}{4} \sum_{i=1}^{n} \left(D^2_{\frac{H}{10}} H\right)_i}$	$\dfrac{\sum_{i=1}^{n} f_i \left(D^2_R H\right)_i}{\sum_{i=1}^{n} \left(D^2_R H\right)_i} = \dfrac{\sum_{i=1}^{n} V_i}{\frac{\pi}{4} \sum_{i=1}^{n} \left(D^2_R H\right)_i}$
(3)	$\frac{1}{n} \sum_{i=1}^{n} f'_i = \frac{1}{n} \sum_{i=1}^{n} \dfrac{V_i}{\frac{\pi}{4} \left(D^2_{\frac{H}{10}} H\right)_i}$	$\frac{1}{n} \sum_{i=1}^{n} f_i = \frac{1}{n} \sum_{i=1}^{n} \dfrac{V_i}{\frac{\pi}{4} \left(D^2_R H\right)_i}$
	$D_{\frac{H}{10}}$ = diameter at height $\frac{1}{10} H_{tot}$ H = total height	D_R = reference diameter H = total height

Each of these 3 formulas is a weighted mean of the individual form factors, the weight given to a tree being $(D^2H)^2$ *in (1) and* (D^2H) *in (2) ; in (3), the trees are given equal weights. The formula to be taken depends on the relationship between the variance of* V *and* D^2H *, with the following rule : if the variance of* V *is proportional to* $(D^2H)^\alpha$ *, the variance of the form factor is then proportional to* $(D^2H)^{\alpha-2}$ *the weight to be given to a tree is* $(D^2H)^{2-\alpha}$.

Formula (1) is thus valid if the variance of V *does not depend on* D^2H ($\alpha = 0$)*; formula (2) is valid if the variance of* V *is proportional to* D^2H ($\alpha = 1$) *and formula (3) is valid if the variance of* V *is proportional to* $(D^2H)^2$ ($\alpha = 2$) .

In practice, it is thus necessary to start by estimating α *; the same problem occurs before fitting a volume equation by regression as will be explained in paragraphs 353.14 and 353.2. The* α *coefficient is often found to lie between 1 and 2 : formulas (2) or (3) are therefore the most used ; in absence of no precise knowledge about the law of variation of the variance of* V *with* α *, it is recommended to take formula (2) ; moreover, this formula is the most practical since the total volume of the set of trees,* $\sum_{i=1}^{n} V_i$ *, appears explicitly in it.*

242 Description of stem form by the equation of the taper curve

242.1 The two types of curve - Problems of scaling

Having measured diameter at different heights, data can be represented by two types of diagram :

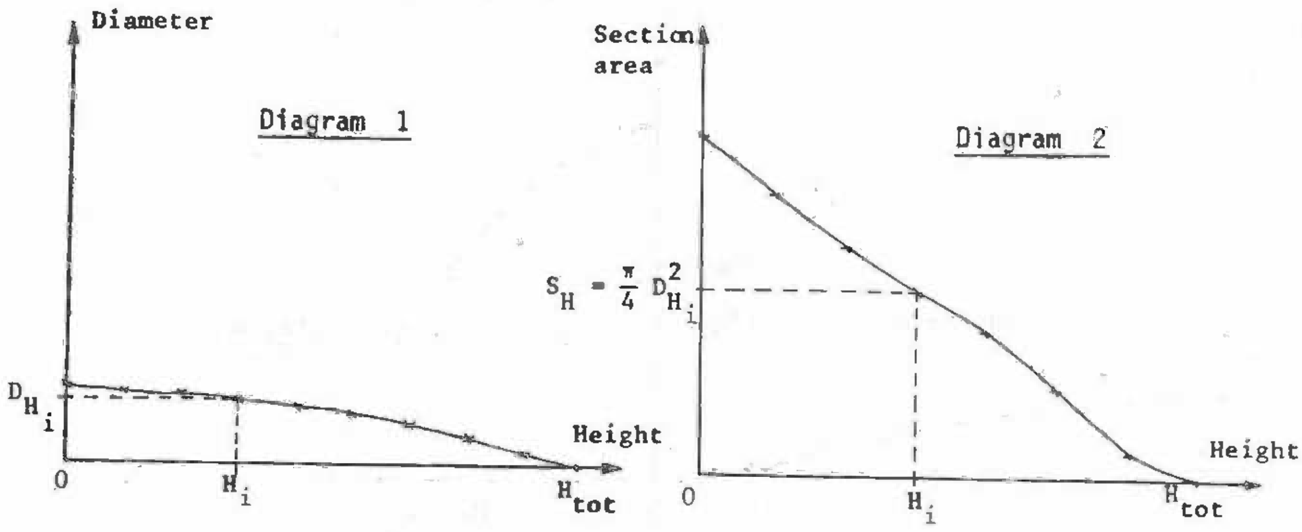
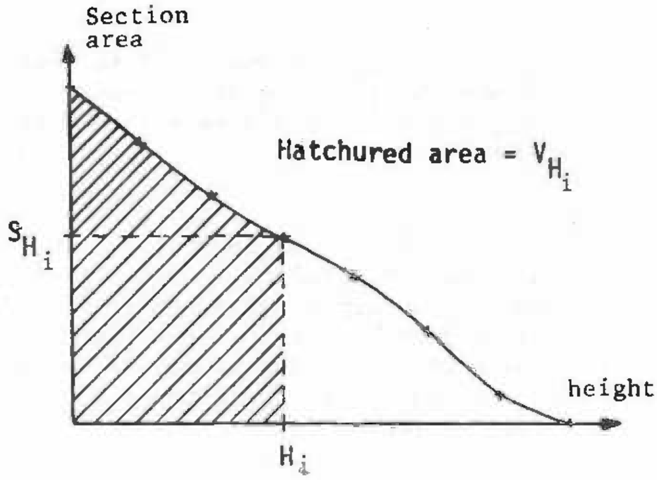
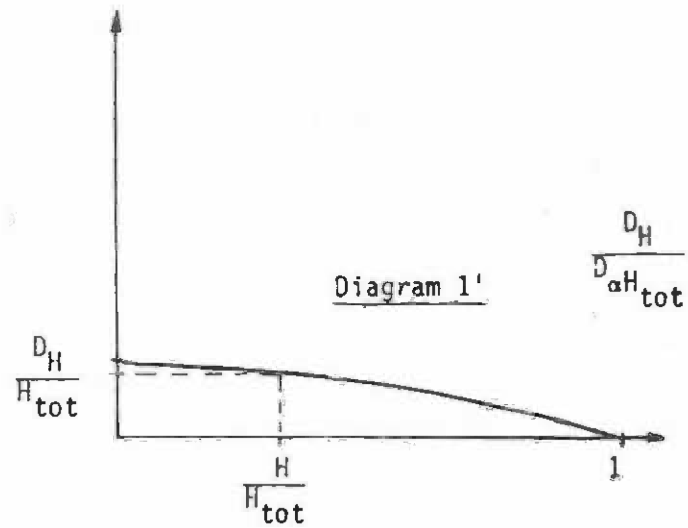


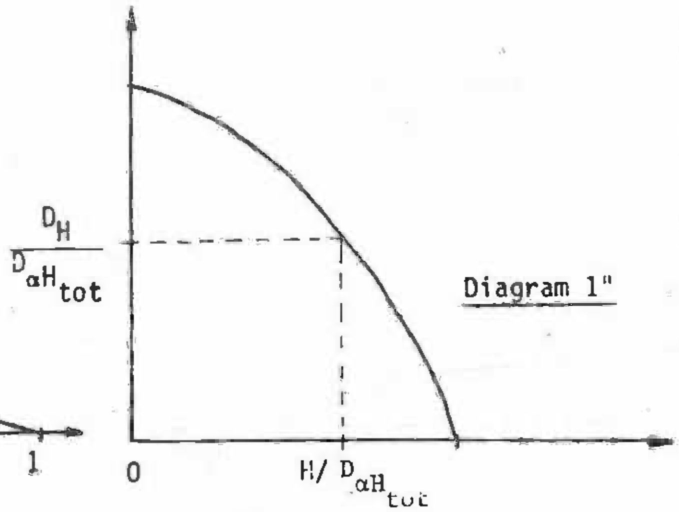
Diagram 1 represents the stem as it is seen ; diagram 2 offers the advantage of showing the volume V_{H_i} *to a given height* H_i.



In order that two trees of same form be represented by the same curve and that two trees with the same curve have the same form, the scales have to be changed.

Here are two possibilities for type 1 diagram :



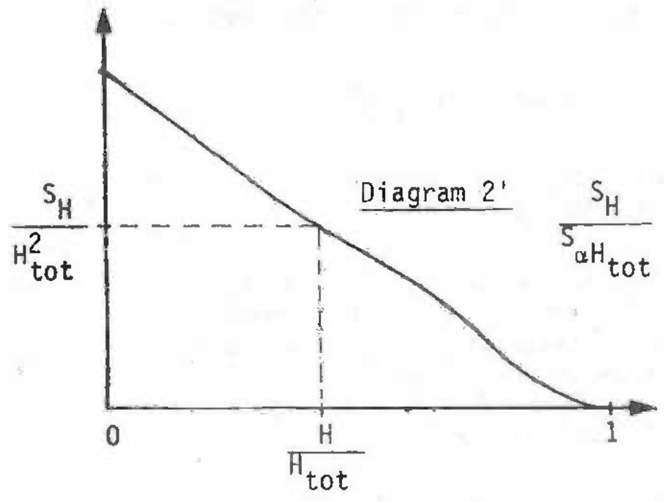


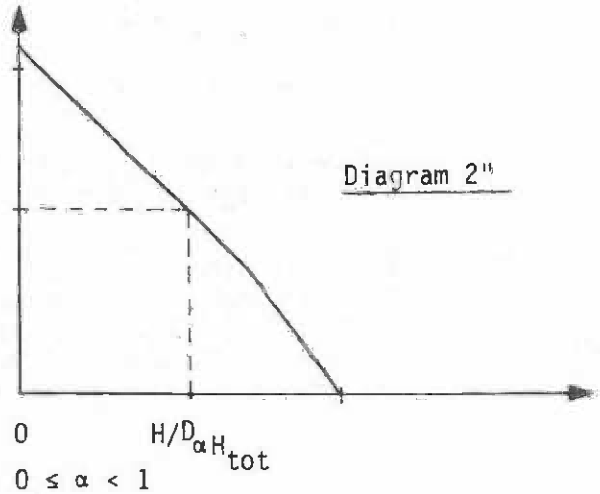
$0 \leq \alpha < 1$

Example :

$\alpha = \frac{1}{10} \rightarrow D_{\alpha H_{tot}}$ = diameter at $\frac{1}{10} H_{tot}$

and the two corresponding possibilities for type 2 diagram :





$0 \leq \alpha < 1$

Example :

$$\alpha = \frac{1}{10} \rightarrow \begin{cases} D_{\alpha H_{tot}} = \text{diameter at } \frac{1}{10} H_{tot} \\ S_{\alpha H_{tot}} = \text{area of section at } \frac{1}{10} H_{tot} \end{cases}$$

Whatever the heights of diameter measurements are, diagrams 1' and 2' can be done ; diagrams 1" and 2" can be done only when diameters have been taken at the same relative heights on each tree.

242.2 Fitting a taper curve by calculation

242.21 Principle

The interest in having a formula for the taper curve is to allow an easy volume calculation of the stem portion between two heights H_1 *and* H_2.

It is natural to consider first the model :

$$S_H = b_0 + b_1 H + b_2 H^2 + b_3 H^3 \qquad \textit{(see example 242.221)}$$

because we have seen in paragraph 231 that this relation is satisfied by most of the simple geometrical solids to which a stem can be compared.

If this formula describes the form badly, here are two possible solutions :

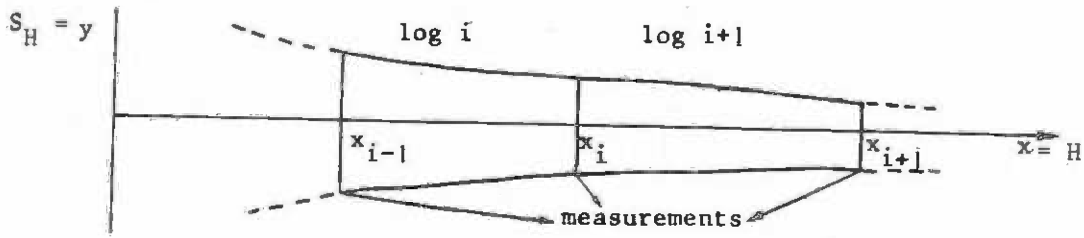
(i) *try a polynomial of higher degree ; the model is thus written in the general form :*

$$S_H = b_0 + b_1H + b_2H^2 + \dots + b_pH^p = \sum_{k=0}^{p} b_k H^k$$

(since this model has p+1 *parameters, at least* (p+1) *measurements* (H, S_H) *are necessary to fit it).*

(ii) *divide the stem into logs and fit a model to each of them, with constraints on the coefficients to force the curves to join correctly. There are numerous ways to proceed, according to the number of measurements available, the number of logs considered, the models chosen for each log and the conditions imposed for the junction of the curves.*

The example of § 242.222 gives a case where 4 measurement points are available and 2 logs are considered. The method of "cubic spline functions" *(see BONEVA et al. ref. 1) is the extreme case of functions of this type because the logs are defined by two successive diameter measurements :*



To each log a cubic is fitted :

log i : $\hat{y} = a_i + b_ix + c_ix^2 + d_ix^3$

log i+1 : $\hat{y} = a_{i+1} + b_{i+1}x + c_{i+1}x^2 + d_{i+1}x^3$

The coefficients of the cubic are obtained by writing that, at each measurement point, $\hat{y}$ *equals* y *and its first and second derivatives equal the first and second derivatives of the cubic of the adjacent log :*

$$a_i + b_i x_i + c_i x_i^2 + d_i x_i^3 = y_i$$

$$a_i + b_i x_{i-1} + c_i x_{i-1}^2 + d_i x_{i-1}^3 = y_{i-1}$$

$$b_i + 2c_i x_i + 3d_i x_i^2 = b_{i+1} + 2c_{i+1} x_i + 3d_{i+1} x_i^2$$

$$c_i + 3d_i x_i = c_{i+1} + 3d_{i+1} x_i$$

Knowing the coefficients of the cubic for log i+1, *this system of 4 equations can be solved, which gives values of* a_i, b_i, c_i, d_i.

The solution is thus obtained step by step : the form of the last log is imposed (conic form for instance) ; the cubic of the preceding log is derived and so on until the first log (the symmetric procedure can be followed, fixing the form of the first log and deriving the cubics step by step from bottom to top of the tree).

Remarks ;

- *A taper curve can be fitted to the bole only ; what has been said is still valid provided that bole height takes place of total height in the models ; a simple model (cubic) is sufficient in general.*
- *Instead of the area of the section, the diameter can be expressed as a function of height. In general, the model which is used is also a polynomial :*

$$D_H = \sum_{k=0}^{p} c_k H^k$$

- *Taking account of the previous paragraph, the models are transformed in order to get coefficients which are characteristic of the form :*

$$y = \sum_{k=0}^{p} b'_k x^k \quad ; \quad y = \frac{S_H}{H_{tot}^2} \qquad x = \frac{H}{H_{tot}}$$

$$y = \sum_{k=0}^{p} b''_k x^k \quad ; \quad y = \frac{S_H}{S_{\alpha H_{tot}}} \qquad x = \frac{H}{D_{\alpha H_{tot}}}$$

$$(0 \leq \alpha < 1)$$

$$y = \sum_{k=0}^{p} c'_k x^k \quad ; \quad y = \frac{D_H}{H_{tot}} \qquad x = \frac{H}{H_{tot}}$$

$$y = \sum_{k=0}^{p} c''_k x^k \quad ; \quad y = \frac{D_H}{D_{\alpha H_{tot}}} \qquad x = \frac{H}{D_{\alpha H_{tot}}}$$

$$(0 \leq \alpha < 1)$$

Variants can be brought to these models (see ref. 6 - 9 - 13).

- *Formula to calculate a volume knowing a taper curve.*

Let us suppose that the formula of the curve is :

$$S = b_0 + b_1 H + \dots + b_p H^p$$
↓
area of section
at height H.

The integral of S *is* $g(H) = b_0 H + b_1 \frac{H^2}{2} + \dots + b_p \frac{H^{p+1}}{p+1}$

The volume of the section of stem between heights H_1 *and* H_2 *is* $g(H_2) - g(H_1)$.

If the taper curve is given by a function relating diameter to height :

$$D = c_0 + c_1 H + \dots + c_p H^p$$

the expression of $S = \frac{\pi}{4} D^2$ *has to be written first ; the required volume is* $g(H_2) - g(H_1)$ *where* $g(H)$ *is the integral of* S *; calculations are more numerous and less precise ; this is why it is better to express a taper curve as* $S = f(H)$ *instead of* $D = f(H)$.

To calculate the volume until a given crosscut, the same method is followed after having calculated the height of this crosscut. This is easy by looking at the plotted curve but, with a computer, the calculation raises problems which cannot be treated here.

- *It is necessary to verify that the taper curve which is obtained is realistic :* y *should not get negative and should decrease as* x *increases.*

242.22 Examples

242.221 Four measurement points - Fitting a 3^d^ degree polynomial

Let us take again the tree of example 233.2 and try to fit the model : $y = a + bx + cx^2 + dx^3$; $y = \frac{S_H}{H^2_{tot}}$ $\quad x = \frac{H}{H_{tot}}$

The solution of the following system :

$$\begin{cases} a + b\,\frac{0.2}{9.5} + c\left(\frac{0.2}{9.5}\right)^2 + d\left(\frac{0.2}{9.5}\right)^3 = \frac{\pi}{4}\left(\frac{0.35}{9.5}\right)^2 \\ a + b\,\frac{1.3}{9.5} + c\left(\frac{1.3}{9.5}\right)^2 + d\left(\frac{1.3}{9.5}\right)^3 = \frac{\pi}{4}\left(\frac{0.31}{9.5}\right)^2 \\ a + b\,\frac{4}{9.5} + c\left(\frac{4}{9.5}\right)^2 + d\left(\frac{4}{9.5}\right)^3 = \frac{\pi}{4}\left(\frac{0.16}{9.5}\right)^2 \\ a + b + c + d = 0 \end{cases}$$

gives : $a = 1.1035 \times 10^{-3}$

$b = -1.7385 \times 10^{-3}$

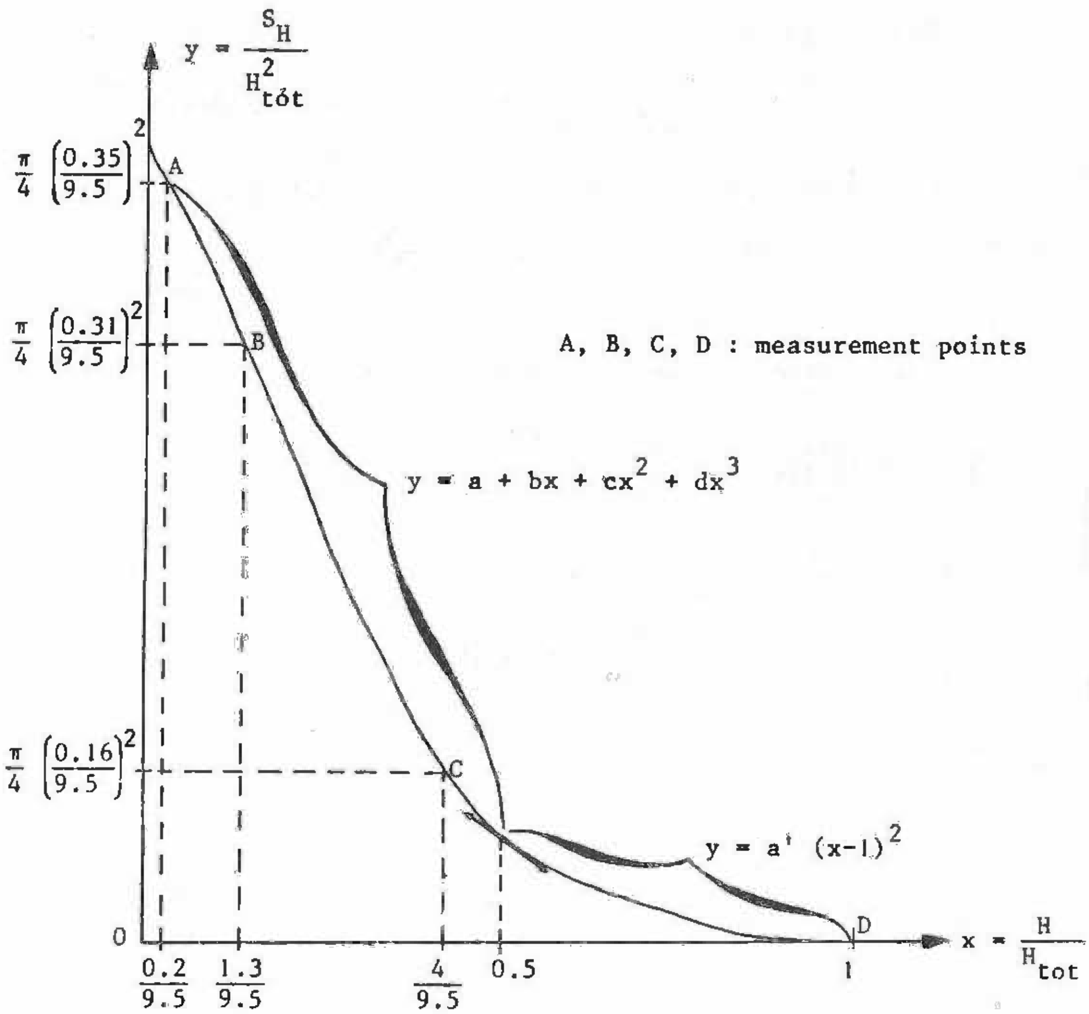
$c = -1.9100 \times 10^{-3}$

$d = 2.5450 \times 10^{-3}$

This curve cannot be retained since y *is negative when* x *is between 0.55 and 1.*

242.222 Four measurement points. Division into two logs and fitting a curve to each of them

Let us take again the previous tree and suppose that the upper half of the stem is a cone ; let us fit a 3^d^ degree polynomial to the lower half, with the constraint that the two curves are tangent at the junction-point.



The system to be solved is :

$$\begin{cases} a + b\,\frac{0.2}{9.5} + c\left(\frac{0.2}{9.5}\right)^2 + d\left(\frac{0.2}{9.5}\right)^3 = \frac{\pi}{4}\left(\frac{0.35}{9.5}\right)^2 & \text{: passing through A} \\ a + b\,\frac{1.3}{9.5} + c\left(\frac{1.3}{9.5}\right)^2 + d\left(\frac{1.3}{9.5}\right)^3 = \frac{\pi}{4}\left(\frac{0.31}{9.5}\right)^2 & \text{: passing through B} \\ a + b\,\frac{4}{9.5} + c\left(\frac{4}{9.5}\right)^2 + d\left(\frac{4}{9.5}\right)^3 = \frac{\pi}{4}\left(\frac{0.16}{9.5}\right)^2 & \text{: passing through C} \\ a + b\,0.5 + c\,(0.5)^2 + d\,(0.5)^3 = a'\,(0.5-1)^2 \\ b + 2c\,0.5 + 3d(0.5)^2 = 2a'(0.5-1) \end{cases}$$

in the point with abscissa 0.5 the two curves join and are tangent

one gets : $a = 1.0976 \times 10^{-3}$ $\quad d = 7.4226 \times 10^{-3}$

$b = -1.4004 \times 10^{-3}$ $\quad a' = 0.5671 \times 10^{-3}$

$c = -4.7336 \times 10^{-3}$

The corresponding curve is drawn above; the fit is correct.

242.223 Description of the mean profile of 3 stems with a 3^d degree polynomial

Diameters at different heights have been measured on 3 trees (H in meters and D in centimeters) :

Tree 1		Tree 2		Tree 3	
H	D_H	H	D_H	H	D_H
0.2	44	0.3	57	0.3	66
1.2	39	1.3	52	1.3	61
1.3	38	2.3	47	2.3	56
2.2	34	3.3	43	3.3	52
3.2	31	4.3	39	4.3	48
4.2	27	5.3	36	5.3	44
5.2	25	6.3	33	6.3	41
6.2	22	7.3	31	7.3	38
7.2	20	8.3	29	8.3	36
8.2	17	9.3	26	9.3	34
9.2	12	10.3	23	10.3	32
9.8	7	11.3	19	11.3	29
10	0	12.3	13	12.3	25
		12.8	7	13.3	21
		13	0	14.3	14
				14.8	7
				15	0

Do these stems have the same form ?

The values of $\frac{H}{H_{tot}} = x$ *and* $\frac{\pi}{4}\left(\frac{D_H}{H_{tot}}\right)^2 = y$ *are calculated with heights and diameters in meters.*

Tree 1		Tree 2		Tree 3	
x	$y \times 10^4$	x	$y \times 10^4$	x	$y \times 10^4$
0.02	15.205	0.023	15.099	0.020	15.205
0.12	11.946	0.100	12.566	0.087	12.989
0.13	11.341	0.177	10.266	0.153	10.947
0.22	9.079	0.254	8.593	0.220	9.439
0.32	7.548	0.331	7.069	0.287	8.042
0.42	5.726	0.408	6.023	0.353	6.758
0.52	4.909	0.485	5.061	0.420	5.868
0.62	3.801	0.562	4.466	0.487	5.041
0.72	3.142	0.638	3.908	0.553	4.524
0.82	2.270	0.715	3.142	0.620	4.035
0.92	1.131	0.792	2.458	0.687	3.574
0.98	0.385	0.869	1.678	0.753	2.936
1	0	0.946	0.785	0.820	2.182
		0.984	0.228	0.887	1.539
		1	0	0.953	0.684
				0.987	0.171
				1	0

Plotting these values shows that the relation between y *and* x *is practically the same in each tree. The 3 trees can thus be considered of same form. Let us see if the model*

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

describes well the common form.

The coefficients have been calculated by the method described in appendix A, (§ A 1.4) : solution of the system of 4 equations with 4 unknowns obtained by forcing the curve to pass through the four following points :

x	0	0.3	0.675	1
y	0.0016	0.000766	0.000350	0

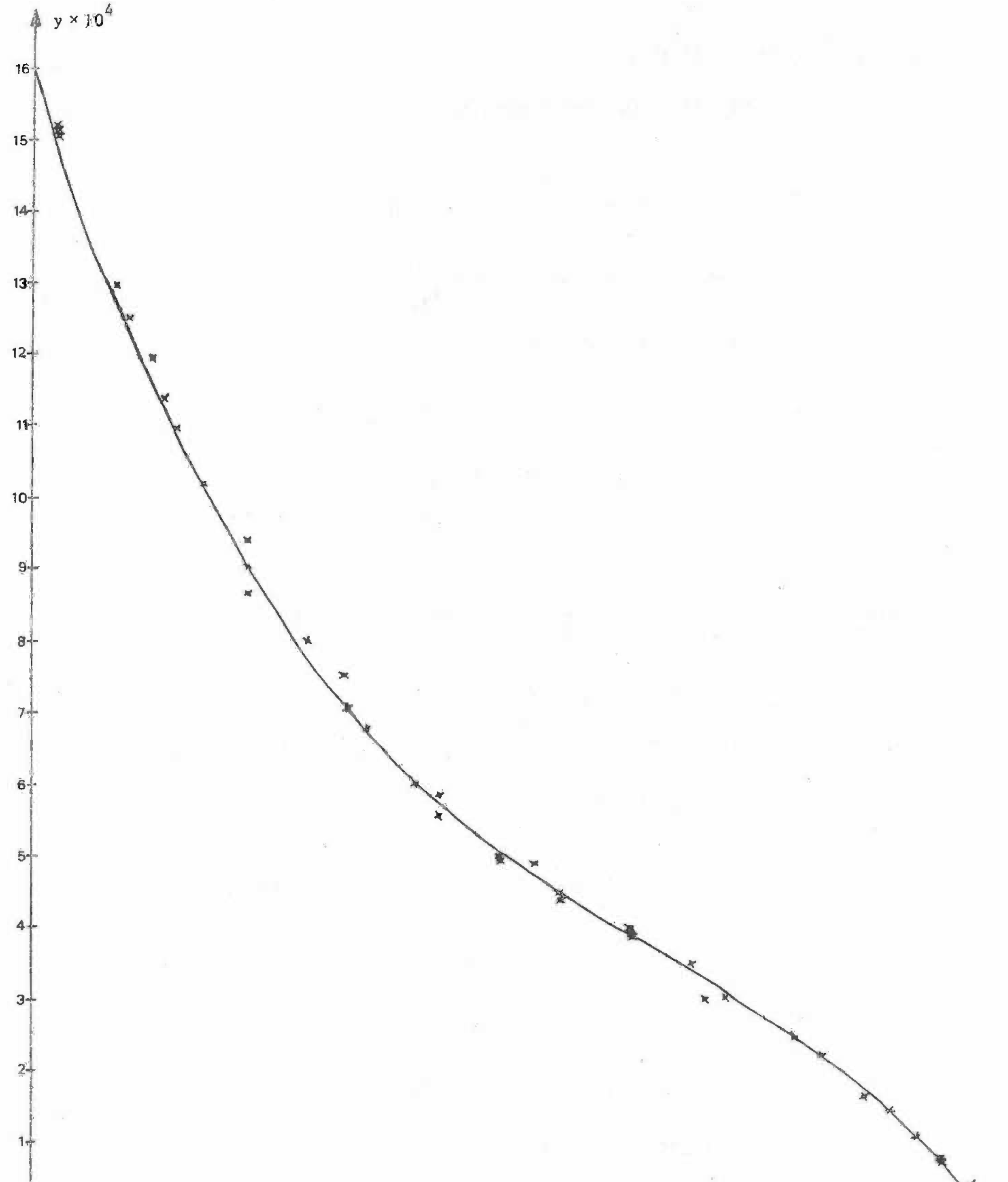
The result is : $a_0 = 16 \times 10^{-4}$

$a_1 = -40.1434 \times 10^{-4}$

$a_2 = 48.4310 \times 10^{-4}$

$a_3 = -24.2876 \times 10^{-4}$

The graph of the corresponding curve shows that the fit is good.


y × 10^4
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x

Example of volume calculation :

The formula of the taper curve is :

$$\frac{S}{H_{tot}^2} = a_0 + a_1 \frac{H}{H_{tot}} + a_2 \frac{H^2}{H_{tot}^2} + a_3 \frac{H^3}{H_{tot}^3}$$

$$\Longrightarrow S = a_0 H_{tot}^2 + a_1 H_{tot} H + a_2 H^2 + \frac{a_3}{H_{tot}} H^3$$

The integral of S *is :*

$$g(H) = a_0 H_{tot}^2 H + a_1 H_{tot} \frac{H^2}{2} + a_2 \frac{H^3}{3} + \frac{a_3}{H_{tot}} \frac{H^4}{4}$$

$$\Longrightarrow g(H) = \left(a_0 x + \frac{a_1}{2} x^2 + \frac{a_2}{3} x^3 + \frac{a_3}{4} x^4 \right) H_{tot}^3 \quad \textit{with} \quad x = \frac{H}{H_{tot}}$$

The volume of the stem between heights H_1 *and* H_2 *is* $g(H_2) - g(H_1)$ *; for example, the total volume of the stem is :*

$$V_{tot} = g(H_{tot}) - g(H_{stump})$$

$$g(H_{tot}) = \left(a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4} \right) H_{tot}^3 = 6.00007 \times 10^{-4} \times H_{tot}^3$$

Let us suppose that $\frac{H_{stump}}{H_{tot}} = \frac{1}{100}$:

$$g(H_{stump}) = \left(a_0\, 10^{-2} + \frac{a_1}{2}\, 10^{-4} + \frac{a_2}{3}\, 10^{-6} + \frac{a_3}{4}\, 10^{-8} \right) H_{tot}^3$$

$$= 15.80089 \times 10^{-6} \times H_{tot}^3$$

Thus : $V_{tot} = 5.84206 \times 10^{-4} \times H_{tot}^3$ *with* H_{tot} *in meters and* V_{tot} *in* m^3.

The total volume of the stem of a tree with total height 14 m. is therefore :

$$V_{tot} = 5.84206 \times 10^{-4} \times (14)^3 = 1.6031\ m^3$$

what is the "bigwood" volume of that stem ? The height H_7 *where is* *located the 7 cm diameter crosscut is such that :*

$$\frac{\frac{\pi}{4}(0.07)^2}{14^2} = a_0 + a_1 x_7 + a_2 x_7^2 + a_3 x_7^3 \qquad \left(x_7 = \frac{H_7}{14} \right)$$

By successive approximations starting from the value x = 0.987 *read on the curve, we get* $x_7 = 0.987608$ *; if we replace* x_7 *by this value and* H_{tot} *by* 14 *in the expression for* g(H) , *we get :*

$g(\tilde{H}_7) = 1.64605\ m^3$.

Now, $g(H_{stump}) = 15.80089 \times 10^{-6} \times (14)^3 = 0.04336\ m^3$

the "bigwood" stem volume is therefore : $1.64605 - 0.04336 = 1.6027\ m^3$.

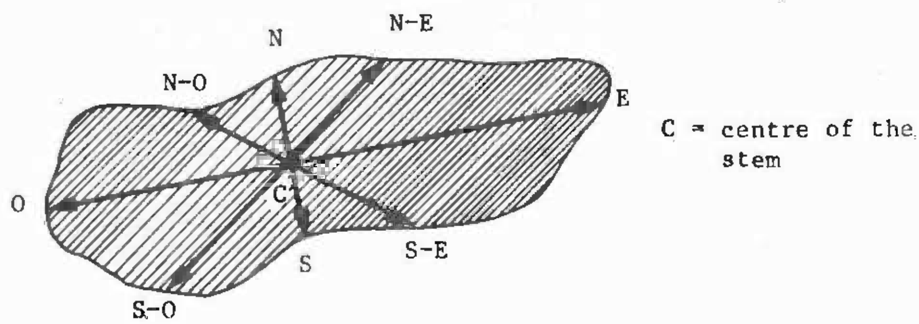
243 Crown measurements

A complete description of tree form includes measurements on crown : these measurements are possible only if the crown is entirely visible.

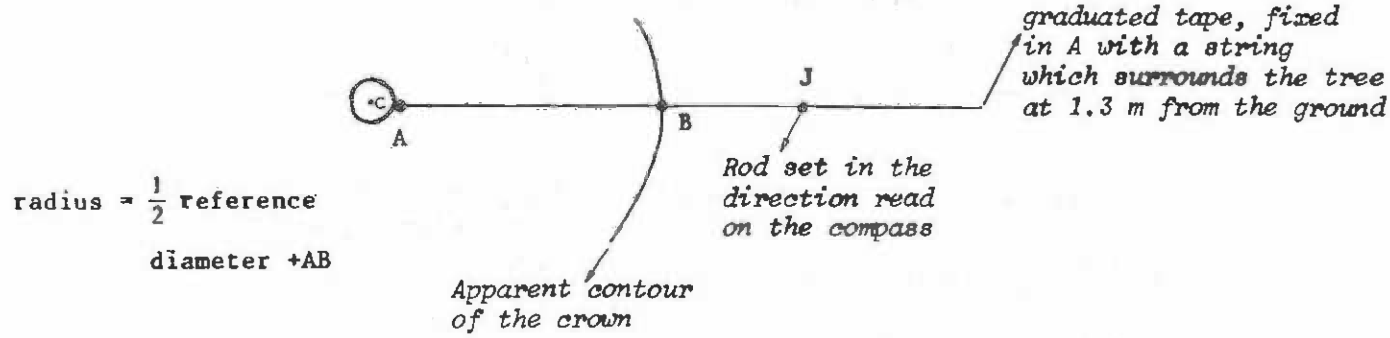
Height : *distance between the end of the bole and the tip of the tree ; it is measured with a dendrometer, as the difference between two measurements.*

Measurements on the horizontal projection : *To describe correctly the projection of the crown on a horizontal plane, the number of radii* should be more than one as this *projection differs from a circle : at least 4, preferably 8, radii are measured, in directions forming equal angles :*

Example :



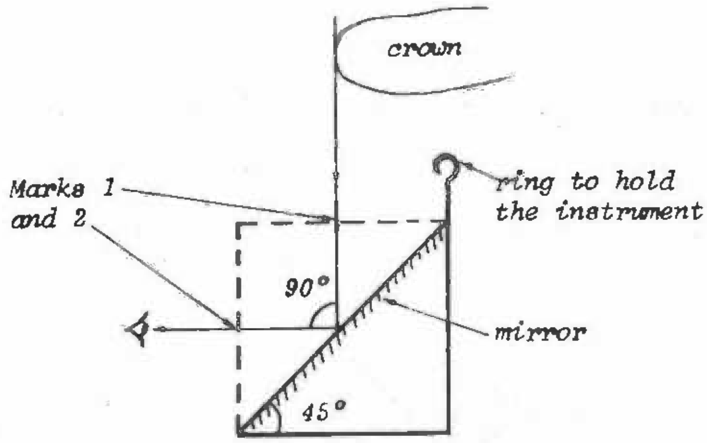
Procedure to measure the radius in one direction :



Walk forwards and backwards on AJ *line and locate point* B *with an instrument. Here are two examples of such instruments :*

a/ A mirror-type instrument

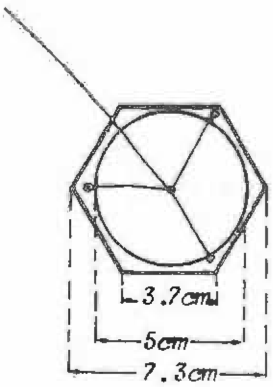
Description : *The instrument is balanced and remains vertical. It contains a plane mirror making an angle of 45° with the horizontal and two panes of glass in the middle of which are two lines : marks 1 and 2. The observer stands so that the two marks come to coincide. It remains to have the point of the contour of the crown in coincidence with these two marks ; the projection of this point on the ground is given by the plumbed line which is fixed on the instrument.*



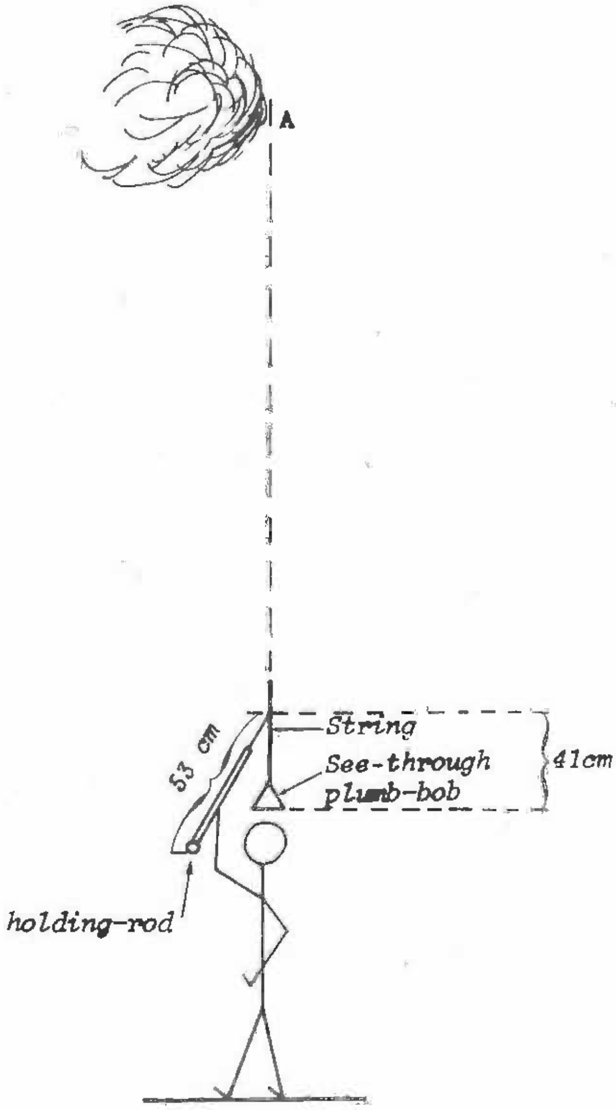
This instrument is difficult to use when the tree touches a neighbour because the mirror allows to see a small part only of the crown and it is difficult to differentiate the branches of a tree from the branches of the other. Its use can be very tiring and time consuming.

b/ The PUN-CHUN crown-meter (*)

Description : *The components of this instrument are a holding-rod, a string and a see-through plumb-bob. Contrary to the mirror-type instrument, the crown-meter allows to see a large part of the crown and therefore to locate far more quickly point A. It is also very simple and can be constructed very cheaply.*



Plumb-bob seen from above



(*) *WAHEED KHAN M.A. (1971) - Pun-Chun crown meter - Indian Forester 96 - n° 6 - pages 332-337*

Quantities calculated with these measurements :

(i) $$S_{crown} = \pi \frac{\sum_{i=1}^{n} r_i^2}{n}$$

r_i = radius in direction i

n = 4 or 8 : number of radii measured

S_{crown} ↓ *area of crown's horizontal projection*

(ii) $$D_{crown} = \sqrt{\frac{4}{\pi} S_{crown}} = 2\sqrt{\frac{\sum_{i=1}^{n} r_i^2}{n}}$$

D_{crown} ↓ *diameter of the crown*

(iii) $$VB_{crown} = \frac{1}{3} S_{crown} \times H_{crown}$$

VB_{crown} ↓ *bulk volume of the crown*

H_{crown} ↓ *height of the crown*

Remark 1 : *These characteristics of crown are important in growth studies but they are rarely taken into account because of the difficulties of field measurements.*

Remark 2 : *The ratio* $$\frac{V_{crown} = \text{volume of wood contained in the crown}}{VB_{crown} = \text{bulk volume of the crown}}$$

is a number less than 1, similar to the piling coefficient, which can be used to estimate V_{crown} *from* VB_{crown} *for a standing tree. It is recommended, each time a tree is felled to measure* V_{crown} *, to measure before felling* H_{crown} *and* S_{crown} *and to derive* VB_{crown}.

3 INDIRECT MEASUREMENT OF A STAND VOLUME : THE TARIFFS (= Volume tables)

31 PRINCIPLE AND DEFINITIONS

To estimate the volume of a stand, one can measure directly the volume of each tree and add all these figures. On large stands this is unacceptable.

A tariff is a table, a formula or a graph, which gives an estimate of the volume of a tree or of a collection of trees from variables called the entries of the tariff.

The entries of the tariff are measurements of the tree (reference - diameter, total height,...) or of the stand (basal area per ha, mean height,...) more easily obtainable than the volume itself.

A tree-tariff gives the volume of a tree from the entries relating to the tree. A stand-tariff gives the volume of a stand directly from the entries relating to the stand itself.

A tree tariff cannot estimate the volume of a single tree with a good precision. Such a tariff is mainly used to estimate the volume of a collection of trees as the sum of volumes of individual trees.

Examples of tree-tariffs :

- *one entry* (D)
  - (i) $V = a + bD + cD^2 + dD^3$
    particular cases : $V = a + bD^2$, $V = a + bD + cD^2$, $V = a + bD^2 + cD^3$, ...
  - (ii) $V = a\,D^b$
- *two entries* (D and H)
  - (i) $V = a + bH + c\sqrt{D^2H} + dD^2H$
    particular cases : $V = a + bD^2H$, $V = a + bH + cD^2H$, ...
  - (ii) $V = a\,D^b\,H^c$
- *three entries* (D, H, $D_{H/2}$)
  - $V = a\,D^b\,H^c\,D_{H/2}^d$

In these tariffs,

- V = *stem volume (or volume of the stem to a diameter limit)*
- D = *reference-diameter*
- H = *total height*
- $D_{H/2}$ = *diameter at height H/2*

Examples of stand-tariffs :

- *2 entries* (G and H) : $V = a + bG + cH + dGH + eGH^2$

particular cases : $V = a + bGH$

$V = a + bG + cGH, \ldots$

$V = a\, G^b\, H^c$

where
- V = *stem-volume/ha (or volume/ha of stems to a diameter limit)*
- G = *basal area/ha*
- H = *mean height or dominant height.*

- *3 entries* (N_1, N_2, N_3) : $V = a_1N_1 + a_2N_2 + a_3N_3$

where
- V = *over bark fuel wood/ha*
- N_1 = *number per ha of poles of total height < 2 m*
- N_2 = *number per ha of poles of total height between 2 and 6 m*
- N_3 = *number per ha of poles of total height > 6 m*

This last model is well adapted to stands in which the measurement of diameter is more difficult that the measurement of height (multiple stems trees, trees of bad form,...).

Remarks :

(i) . *Some tariffs are of an intermediate type between tree-tariffs and stand-tariffs : they give the volume of a tree as a function of variables relating to the tree and variables relating to the stand where it is located. They are tree-tariffs for which the coefficients are known functions of variables relating to the stand.*

Example : $V = (a + bH_{dom}) + (c + dH_{dom})\, D^2$

where V *is the volume of a tree of diameter* D.

Such a tariff is called a parametrized tree-tariff because it can be considered as a family of tree-tariffs $V = a_i + b_iD^2$, H_{dom} *being the parameter which indicates the tariff to be used for the trees of a given stand. To construct such a tariff, one can fit directly* V *to the 3 variables* H_{dom}, D^2 , D^2H_{dom} *or start by establishing a tariff for each class of* H_{dom} *and deduce the final equation subsequently. On this subject, see appendix A (§ A 1.5 and A 1.6).*

(ii) . *A two entries-tariff is more precise than a one entry-tariff but it is more difficult to use. Therefore, we sometimes derive a one entry tariff from a two entries tariff. The following procedure can be indicated :*

- suppose a two entries tariff V = f (D, H) is available. Measure D and H on approximatively 30 trees,
- then, calculate their volume by V = f (D, H) and construct with these 30 trees a tariff with only one entry (D)

  or : on these 30 trees, establish a law H = g (D) and take V = f (D, g(D)) as the one entry tariff.

(iii) . *A tariff must be considered as a tool to be well maintained. For example, it should be updated with additional data as plantations grow and extend.*

32 CHOICE OF THE ENTRIES

The entries of a tariff should be :

- few and easy to measure in order that the tariff will have a wide range of application and be easy to use,
- strongly correlated with the volume,
- weakly correlated to each other in order that the interest in keeping a variable in the model remains when the others are in it.

In general, no more than two entries are used : the first one is always reference diameter, the second one being diameter at a fixed height (5 m for instance) or at a relative height, or bole height, or total height, or crown diameter ... Among these variables, bole height and diameter at a fixed height are more easy to measure, diameter at a fixed height being often more used than height.

33 PROCEDURE TO ESTIMATE THE VOLUME OF A STAND WITH A TARIFF

With a tree tariff.

- Take a sample of n trees in the stand (see § 34) and measure directly the volume of each. Establish the tariff.
- Measure the variables which are the entries of the tariff on the N-n trees which were not used to construct the tariff.
- Estimate the volume of the stand by

$$\hat{V} = \begin{bmatrix}\text{Sum of volumes} \\ \text{of the n trees}\end{bmatrix} + \begin{bmatrix}\text{Tariff estimate of the volume} \\ \text{of the remaining trees}\end{bmatrix}$$

With a stand tariff.

- In stands similar to the stand under study, measure V (volume per ha) and other characteristics (entries of the tariff) more easily measurable. Establish the tariff.
- In the stand under study, measure the variables which are the entries of the tariff and apply the formula.

In fact, this procedure has often to be changed because it is not possible to establish a tariff for every stand. In practice one very often uses a tariff which has not been established with a sample coming from the stand under study.

This is justified if the relation between the volume and the entries of the tariff is approximately the same in the stand and in the sample used to establish the tariff, so :

. *for a* tree-tariff , *the variability of tree forms must be the same in the stand and in the sample. One should therefore ensure that factors which influence the form of trees have the same variability in the stand and in the sample (variability of genetics, environmental factors, silvicultural treatments, age and tree-size). The more the domain of validity of a tariff is intended to be large, the more the sample used to construct it has to be diversified,*

. *the same occurs for a* stand-tariff : *ensure that the variability of environmental factors, densities of trees, silvicultural treatments are similar in the stand under study and in the sample plots used to construct the tariff.*

34 SAMPLE CHOICE TO CONSTRUCT A TARIFF

341 Tree-tariff

For a single species homogeneous stand, one can consider that between 50 and 100 trees (one entry tariff) and between 80 and 150 trees (2 entries tariff) *are needed. For a large heterogeneous region a separate tariff is established for each homogeneous sub-region. Comparison of these tariffs can lead to pooling some of them : so, examples are given in literature of tariffs constructed with several thousands of trees.*

The number of trees is not the only criteria to consider ; the stands where these trees will be taken and the sample trees in these stands have to be chosen ; here are some recommendations :

- divide the region for which the tariff is to be established into homogeneous compartments (with regard to site conditions, silvicultural status,...)
- divide the compartments into age-classes and follow the rules :

(1) $$\frac{\text{Number of sample trees in the age class of the compartment}}{\text{Total number of sample trees}} = \frac{\text{Area of the age class of the compartment}}{\text{Area of the region}}$$

(2) In an age class of a compartment, take the same number of sample trees in each basal area class

In practice, these principles can be hard to apply because the repartition in age classes can be impossible (planted forests of badly known history, natural untreated forests,...). The following rules will then be used as substitutes.

(1') $$\frac{\text{Number of sample trees in the compartment}}{\text{Total number of sample trees}} = \frac{\text{Area of the compartment}}{\text{Area of the region}}$$

(2') In a compartment, take the same number of sample trees in each basal area class.

Comment on rules (2) and (2') :

What is needed is the mean volume of the trees which have a given value of the entries ($D_{1,3}$, H_{tot} ,...)*: the volume variability raising in general with tree size, it is more useful to measure a big tree than a small one. Rules (2) and (2') aim at avoiding that the majority of trees belong to a small number of size classes. Realize that a random sample taken according to the rule* "one tree taken at random over n " *is not what is wanted.*

For example, a tariff is wanted for high forest trees between diameter 20 cm and 1 meter. The range of basal areas is divided into ten equal classes ; the limits of the corresponding diameter classes are 200 - 369 - 482 - 573 - 651 - 721 - 785 - 844 - 899 - 951 - 1000 (mm) *In each of these classes, a sample of about ten trees will be taken according to a sampling design which covers the whole area.*

Remark on mixed forests :

The number of species in natural forests is often such that it is impossible to establish a tariff for every species. Tariffs for groups of species are thus necessary. How to group species ? The simplest way is to plot data (V and D^2 or D^2H) *and decide with these diagrams.*

342 Stand-tariff

These tariffs are until now less used than tree-tariffs : the experimental knowledge is not sufficient to allow reliable recommendations. Consider what follows as a loose guidance :

- take at least thirty plots,
- area of a plot in ares = H_{dom} in meters with a minimum threshold of 10 ares. (H_{dom} = 20 m → plot of 20 ares (0.2 ha),
  H_{dom} = 9 m → plot of 10 ares (0.1 ha)...)

Example :

A tariff is wanted to give the fuel wood volume of a savannah, the mean height of which is about 6 meters. The second model of paragraph 31 is chosen :

$V = a_1N_1 + a_2N_2 + a_3N_3$ *where* N_i *is the number/ha of trees of height class* i *(observe that coefficient* a_i *is here interpreted as the mean volume of a tree of height class* i*).*

Procedure :

- take at random 30 plots 30 m × 30 m
- before felling, inventory each plot by height class. Identification of species is not compulsory,
- fell and stack each plot,
- fit the model on data (V, N_1, N_2, N_3) of plots.

35 DIFFERENT WAYS TO CONSTRUCT THE TARIFF WITH COLLECTED DATA

351 Direct method

This method seems the most natural at first sight : each entry of the tariff is divided into classes.

For a one entry tariff, calculate mean volume in each class.

For a two entries tariff, set a table by crossing the classes of the two entries, allocate the trees (tree-tariff) or the plots (stand-tariff) to the cells, calculate mean volume in each cell.

... etc ...

The series of volume corresponding to the combinations of explaining variables constitutes the tariff. If necessary, a formula can then be adjusted to these values (see appendix A).

Advantage : *Calculation easy.*

Drawbacks : . *the tariff is very imprecise for the combinations of explaining variables where scarce data are available. The law of variation of the volume can be very irregular.*

. *It is impossible to estimate the precision with which the tariff estimates the volume of a stand.*

352 Graphical methods

In practice, they are easy to apply only for one entry tariffs (see appendix A) : trees (tree-tariff) or plots (stand-tariff) are plotted with volume in y-*axis and the explaining variable (entry of the tariff) in* x-*axis. For some* x *values, a point is placed at the mean value of the volumes which correspond to that x and a continuous curve is drawn by hand across these points (see appendix A).*

Advantage : . *Practically no calculation.*
. *The tariff is* "smooth" *(superiority upon the direct method).*
. *The graphical representation calls attention to outliers. Anomalies like negative volumes are instinctively avoided.*

Drawbacks : . *The result depends on plotter's skilfulness and on his intuitive knowledge of the law variation of volume with explaining variables.*
. *Here also, there is no means to estimate with what precision the tariff estimates the volume of a stand. This is the main drawback.*

353 The statistical method : regression analysis

This method is mainly used, the inconvenience of calculations having diminished with the development of computers.

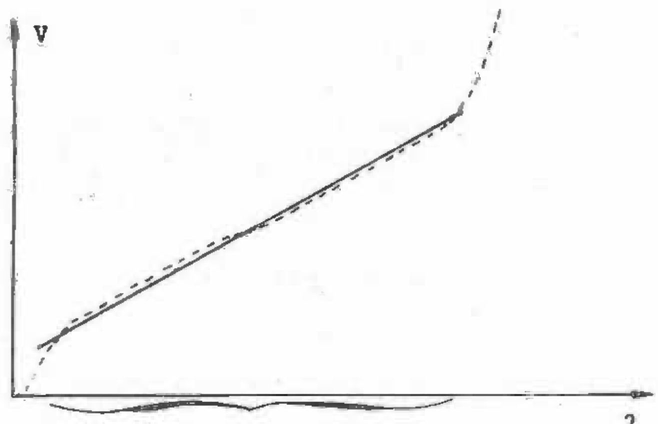
353.1 Concerning the choice of the regression model

Somes examples of models have been given in paragraph 31 and numerous other ones are used ; it is impossible to recommend a unique model (for tree-tariffs for instance, this would mean that for every species in whatever condition, the form factor varies in the same way with D *and* H*). Let us indicate some important points.*

353.11 Simplicity of the model

Always try to have the most simple model as far as possible, that is the one with the fewest number of coefficients. The more numerous the coefficients the more likely V *is to vary illogically with the entries.*

Example :



The ——— *curve is the line* $V = a + bD^2$. *The model :*
$V = a + bD^2 + cD^4 + dD^6 + eD^8$ *can be represented by a curve as strange as the* - - - - - *curve : the two curves will be very close in the usable part of the tariff but a very small extrapolation will be much more dangerous with the complicated model.*

In practice, the following models with two coefficients $V = a + bD^2$, $V = aD^b$, $V = a + bD^2H$, $V = a(D^2H)^b$ *give often good results and have to be tried first.*

First, plot the data :

for a one entry tariff : V and D^2
for a tariff with several entries : V and D^2H ; if data are numerous, plot also V and D^2 for each class of H (or for each class of an other entry). *These diagrams allow a first choice of the regression model and study of relationship between volume variance and entries, in order to decide with which weighting function the model will be fitted.*

353.12 Concerning models where a function of V and not V itself is estimated

Let us take the following model which is very often used ("logarithmic tariff") :

$$V = a\, D^b$$

Least squares estimation of the coefficients consists in seeking a *and* b *which minimize the quantity :*

$$\sum_{i=1}^{n} (V_i - a\, D_i^b)^2$$

This calculation is possible but difficult because the model is not a linear combination of the unknown coefficients. Therefore one takes logarithms and comes down to a linear model :

$$\log V = \log a + b \log D$$

and it is this model which is estimated by least squares ; so, the variable predicted is log V *and not* V. *The* a *and* b *coefficients obtained are such that* $\log a + b \log D$ *is an estimate of the mean of logarithms of the volumes of trees of diameter* D .

The quantity $a\, D^b$ *is therefore an estimate of the geometric (and not arithmetic) mean of the volumes of trees of diameter* D. *Now, logarithmic mean is systematically below arithmetic mean (example : the 4 numbers 3, 4, 7, 10 have a logarithmic mean* $(3 \times 4 \times 5 \times 10)^{1/4} = 5.38$ *and an arithmetic mean* $\frac{1}{4}(3+4+7+10) = 6)$ *; so : a logarithmic tariff systematically underestimates the volume.*

The disadvantage can be partly corrected (see appendix A - § A 2.3) but there is another disadvantage : if one estimates with the tariff the volume of a set of N *trees by :*

$$a \sum_{i=1}^{N} D_i^b$$

the precision of this estimate cannot be known because regression theory gives the precision of :

$$\sum_{i=1}^{N} \log V_i$$

It seems therefore better to use logarithmic tariffs only when it is difficult to fit a simple model where V *appears untransformed.*

353.13 Piecewise fitting of a model

If it is difficult to fit a single model covering the whole range of data (for example, if it is impossible to eliminate bias for small trees and/or big trees), a solution is to divide the range of data and to fit a model in each part.

Let us indicate two ways of acting :

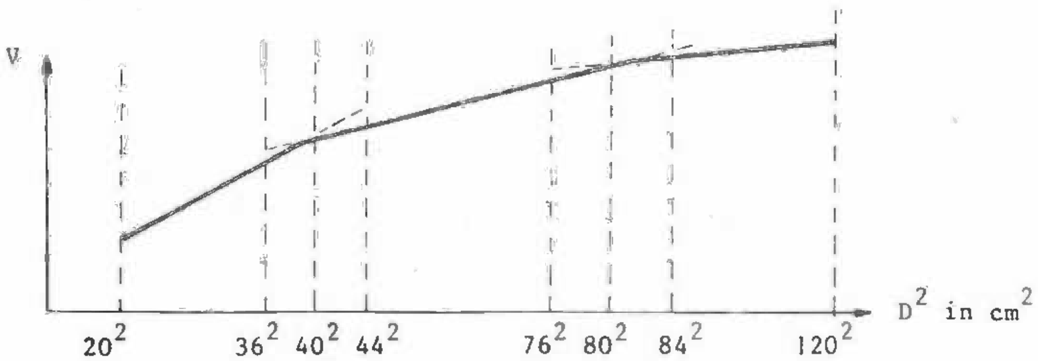
(i) Method with several regression analysis

In order that the models link well, overlapping sub-ranges are taken, with an overlap of about 20 %.

example : *diameters range from 20 to 120 cm*
plotting data shows that :

- *a model* $V = a_1 + b_1 D^2$ *is good for trees with* $20 < D < 40$ *cm*
- *a model* $V = a_2 + b_2 D^2$ *is good for trees with* $40 < D < 80$ *cm*
- *a model* $V = a_3 + b_3 D^2$ *is good for trees with* $80 < D < 120$ *cm.*

The first model will be fitted on trees with $20<D<44$ *cm, the second model on trees with* $36<D<84$*cm and the third model on trees with* $76<D<120$*cm :*

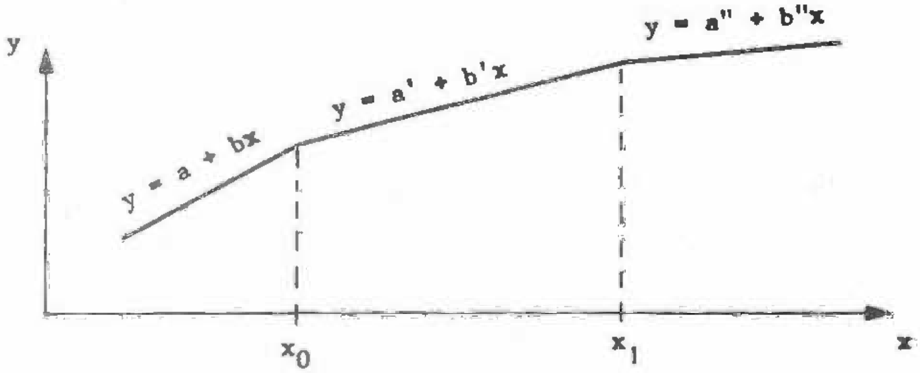


the line ——— *then represents the tariff.*

This procedure offers the advantage of coming down to a series of fittings of simple curves but it has the drawback of not allowing the exact calculus of the residual variance : it is therefore impossible to estimate with which precision the tariff estimates the volume of a set of trees. That is the reason why the second procedure, although a little more complicated as regard the calculations, is rather recommended :

(ii) Method with only one regression analysis

Let us take again the above example :



The values $x_0 = 40$ *and* $x_1 = 80$ *being chosen, the following model with 4 parameters is fitted :*

$$y = a + bz_1 + b'z_2 + b''z_3 \text{ with } z_1 \begin{cases} = x & \text{if } x \leq x_0 \\ = x_0 & \text{if } x \geq x_0 \end{cases}$$

$$z_2 \begin{cases} = 0 & \text{if } x \leq x_0 \\ = x - x_0 & \text{if } x_0 \leq x \leq x_1 \\ = x_1 - x_0 & \text{if } x \geq x_1 \end{cases}$$

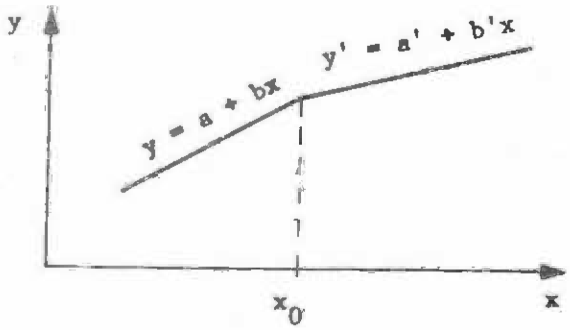
$$z_3 \begin{cases} = 0 & \text{if } x \leq x_1 \\ = x - x_1 & \text{if } x \geq x_1 \end{cases}$$

the following relations give the a' *and* a" *coefficients :*

$a' = a + (b-b')\, x_0$ $\qquad$ $a'' = a' + (b'-b'')\, x_1$

Remarks :

. *Fitting of only two lines :*

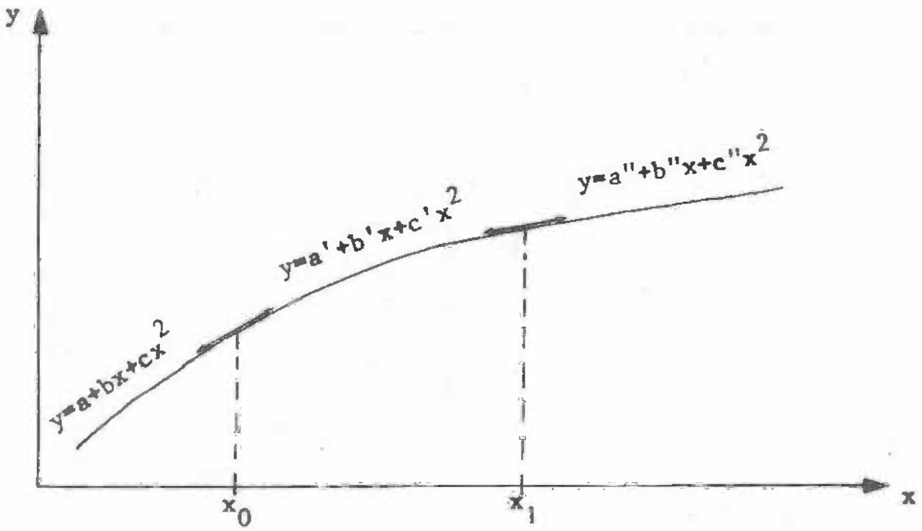


the value x_0 *is chosen and the following model with 3 parameters is fitted :*

$$y = a + bz_1 + b'z_2 \text{ with } \begin{cases} z_1 \begin{cases} = x & \text{if } x \le x_0 \\ = x_0 & \text{if } x \ge x_0 \end{cases} \\ z_2 \begin{cases} = 0 & \text{if } x \le x_0 \\ = x - x_0 & \text{if } x \ge x_0 \end{cases} \end{cases}$$

a' *is given by :* $a' = a + (b-b')\,x_0$

. *To fit three parabolas which are tangent at the junction-points*



fix the x_0 *and* x_1 *values and fit the following model with 5 parameters :*

$$y = a' + b'z_1 + c'z_2 + cz_3 + c''z_4 \text{ with } z_1 = x$$

$$z_2 \begin{cases} = 2xx_0 - x_0^2 & \text{if } x \le x_0 \\ = x^2 & \text{if } x_0 \le x \le x_1 \\ = 2xx_1 - x_1^2 & \text{if } x_1 \le x \end{cases}$$

$$z_3 \begin{cases} = (x_0 - x)^2 & \text{if } x \le x_0 \\ = 0 & \text{if } x_0 \le x \end{cases}$$

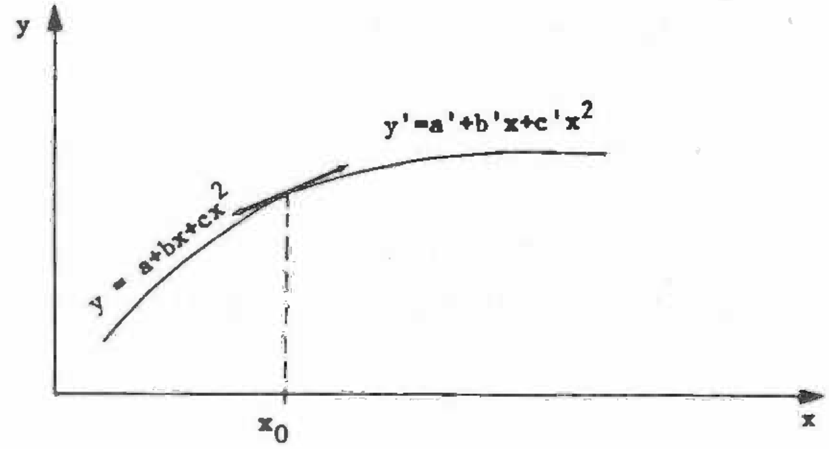
$$z_4 \begin{cases} = 0 & \text{if } x \le x_1 \\ = (x - x_1)^2 & \text{if } x_1 \le x \end{cases}$$

the other parameters are given by :

$$a = a' + (c-c')\,x_0^2 \quad ; \quad b = b' - 2(c-c')\,x_0 \,;$$

$$a'' = a' - (c'-c'')x_1^2 \quad ; \quad b'' = b' + 2(c'-c'')x_1 \,.$$

. *Model to fit only two parabolas which are tangent at the junction-point :*



$$y = a + bz_1 + cz_2 + c'z_3 \text{ with } \begin{cases} z_1 = x \\ z_2 \begin{cases} = x^2 \text{ if } x \le x_0 \\ = x_0\,(2x - x_0) \text{ if } x \ge x_0 \end{cases} \\ z_3 \begin{cases} = 0 \text{ si } x \le x_0 \\ = (x-x_0)^2 \text{ if } x \ge x_0 \end{cases} \end{cases}$$

a' *and* b' *are given by :*

$$a' = a - (c-c')\,x_0^2 \quad ; \quad b' = b + 2(c-c')\,x_0$$

353.14 Weighted or unweighted regression ?

The regression must be fitted with weights when the volume variance depends on the entries. Without entering mathematical explanations, let us say that this is necessary in order to be able to estimate correctly the precision with which the tariff will estimate the volume of a stand ; if the calculation of this precision is not judged necessary and if only a good fit is wanted (that is to say without bias and with small residuals), weighting is not essential.

It is not easy to know the relationship between the variance of volume and the entries ; this necessitates a large amount of data, far greater than is usually available. However, each time very large samples have been collected, the variance of volume has been found to vary, often very much, with the size of the trees. This implies recommending the systematic use of weighted regression. With few data, it is impossible to estimate precisely the weighting function and the following hypothesis are generally taken :

(a_1) volume variance proportional to (D^2) } *one entry tariff*
(a_2) volume variance proportional to $(D^2)^2$ }

(b_1) volume variance proportional to (D^2H) } *two entries tariff*
(b_2) volume variance proportional to $(D^2H)^2$ }

The model is fitted with each hypothesis successively and the best fit is chosen (the one which realizes the best compromise between the following requirements : no bias , small residual standard deviation, simplicity of model). This is done by the program described in the bibliographic reference n° 3 ; it fits the 4 models :

$$V = a + bD^2$$
$$V = a + bD + cD^2$$

under hypothesis (a_1) and (a_2)

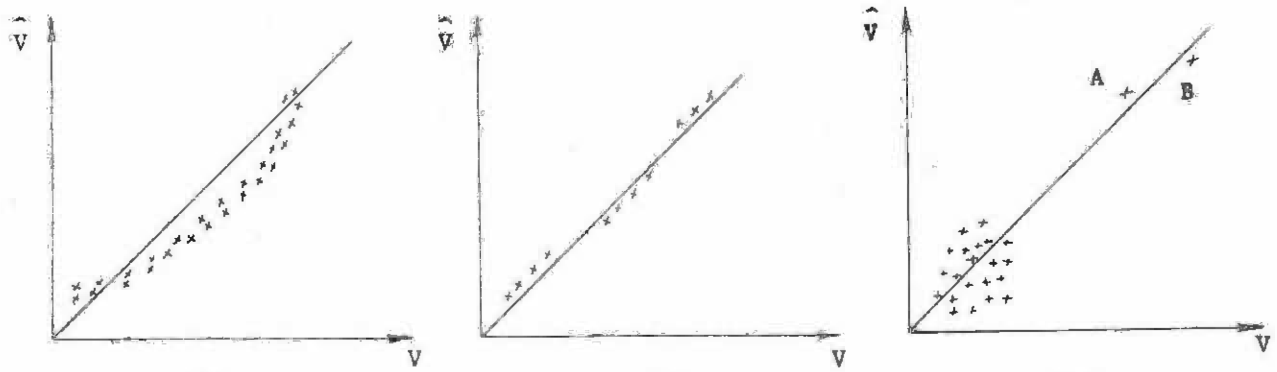
$$V = a + bD^2H$$
$$V = a + b\sqrt{D^2H} + c(D^2H)$$

under hypothesis (b_1) and (b_2)

353.15 How to appreciate the quality of a regression

Never judge of the quality of a regression only by the numerical value of the multiple correlation coefficient R *(correlation coefficient between* V *and* $\hat{V}$ = V adjusted *).*

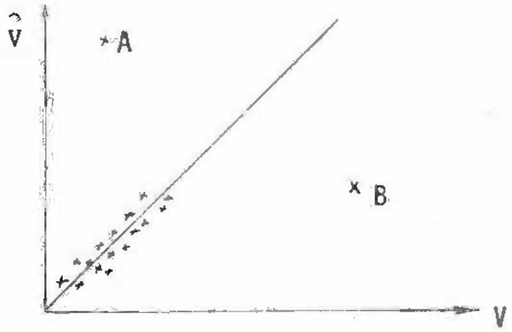
The fit can be bad and R *high : here are three typical situations of such a case :*



(1) (2) (3)

(1) biased model
(2) non homogeneous sample
(3) presence of "abnormal" trees (without trees A and B, R would be low)

It can also happen that the fit is good and R *low ; it can be so for instance when some trees are* "abnormal".



For trees other than A *and* B, *the fit is good. Without* A *and* B, R *would be high.*

Numerous quantities other than R *can be considered : the most use are the residual standard deviation :*

$$s = \sqrt{\frac{\Sigma(r_i - \bar{r})^2}{n - c}} \quad \textit{where} \quad r_i = V_i - \hat{V}_i \,, \quad \bar{r} = \frac{1}{n} \Sigma\, r_i \,,$$

c = number of coefficients in the model
n = number of data (trees)

and the residual coefficient of variation :

$$\frac{s}{\bar{V}} \quad \textit{where} \quad \bar{V} = \frac{1}{n} \Sigma V_i = \text{mean of measured volumes}$$

(for a model with a constant term where V *is untransformed, the numerator of* s *takes the simple form :*

$$\sqrt{\Sigma V^2 - \Sigma \hat{V}^2} \quad \textit{because then} : \Sigma V = \Sigma \hat{V} \quad \textit{and} \quad \Sigma V \hat{V} = \Sigma \hat{V}^2 \;).$$

The "aggregate deviation" :

$$\frac{\Sigma V - \Sigma \hat{V}}{\Sigma \hat{V}}$$

or the "average deviation" :

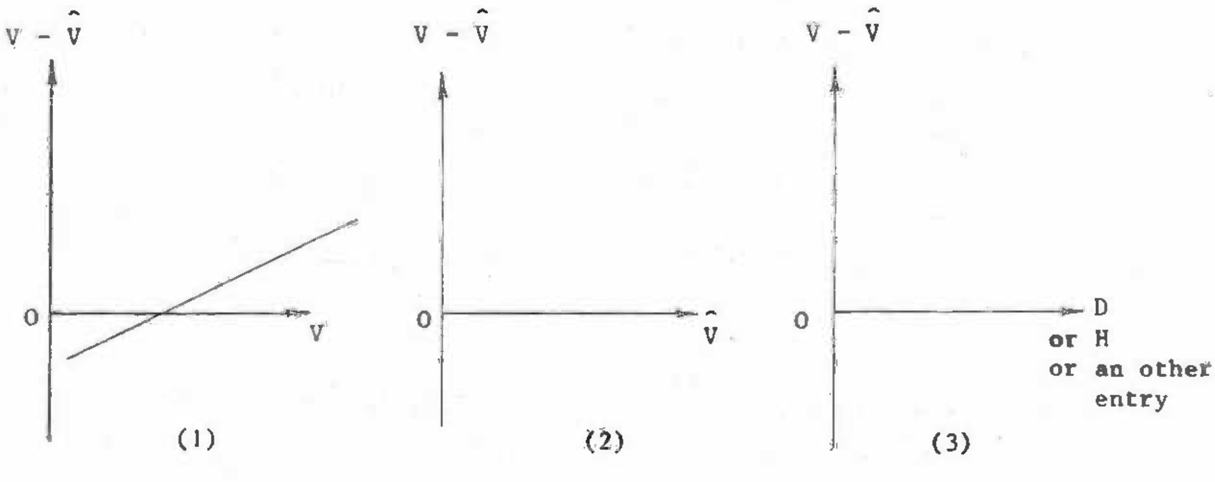
$$\frac{\Sigma |V - \hat{V}|}{\Sigma \hat{V}}$$

are sometimes used also. What has been said about R *can be repeated for each of them : they do not allow one to appreciate completely the quality of the fit.*

To judge effectively of the quality of a regression :

(i) *plot on the same graph the data and the fitted curve. For a one entry tariff, it will be the graph of* V *against* D *; for a two entries tariffs, it is recommended, whatever the model is, to plot* V *against* D^2H.

(ii) *calculate and plot the residuals* $V - \hat{V}$ *; three types of plotting are possible :*



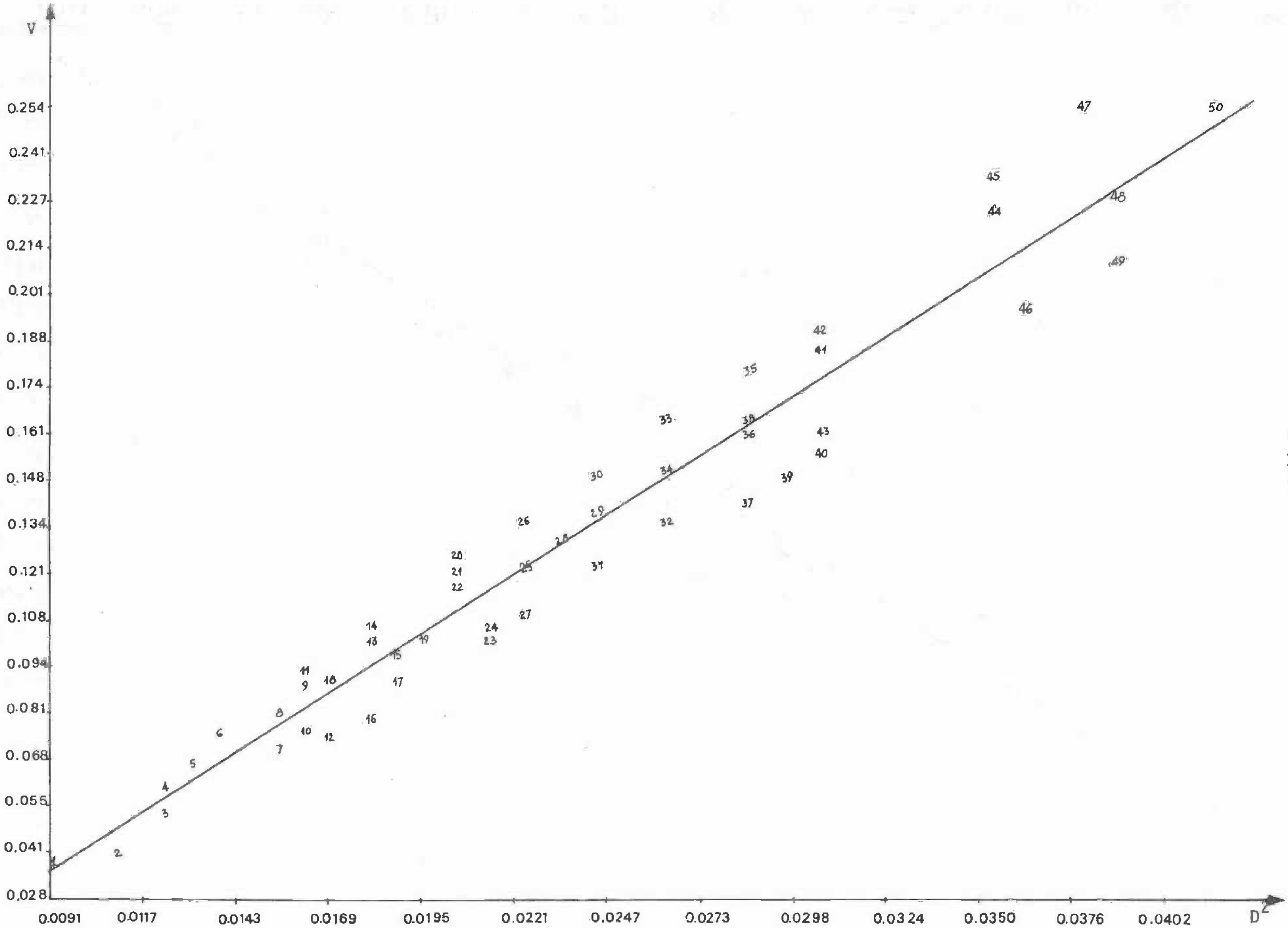
When there is no bias , diagram (1) is well scattered around an upward line (the slope of the regression line of $V - \hat{V}$ *in function of* V *is* $1 - R^2$ *) ; on diagram (2), data are scattered around* $\hat{V}$-axis. *When there is a bias, diagrams (3) provide an aid to decide how to correct the model (see appendix A, § A 2.6).*

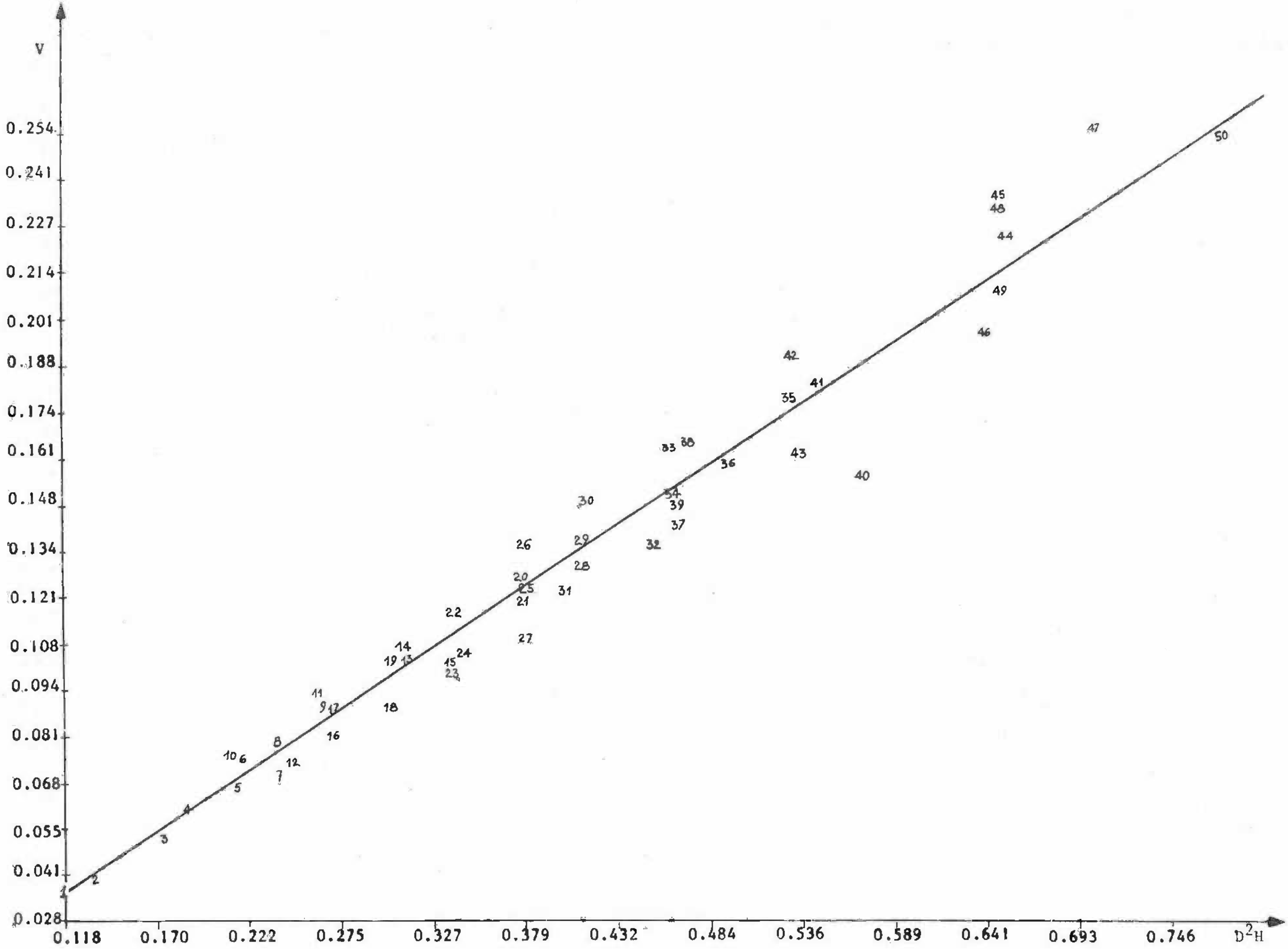
353.2 Example

The reference diameter, the total height and the big wood stem volume have been measured on 50 trees :

Tree n°	D_m	H_m	V_{m3}	Tree n°	D_m	H_m	V_{m3}
1	0.095	12.90	0.037	26	0.150	17.00	0.135
2	0.105	12.00	0.040	27	0.150	16.90	0.108
3	0.111	14.00	0.052	28	0.153	17.50	0.132
4	0.111	15.00	0.060	29	0.156	17.00	0.138
5	0.115	16.50	0.067	30	0.156	17.00	0.148
6	0.118	15.90	0.075	31	0.156	16.30	0.123
7	0.124	15.40	0.070	32	0.162	17.16	0.135
8	0.124	15.30	0.080	33	0.162	17.50	0.165
9	0.127	16.50	0.090	34	0.162	17.50	0.150
10	0.127	13.00	0.075	35	0.169	18.50	0.180
11	0.127	16.05	0.093	36	0.169	17.30	0.160
12	0.131	14.50	0.074	37	0.169	16.30	0.140
13	0.134	17.40	0.102	38	0.169	16.50	0.165
14	0.134	17.10	0.107	39	0.172	15.70	0.148
15	0.137	18.00	0.100	40	0.175	18.50	0.156
16	0.134	15.00	0.080	41	0.175	17.70	0.184
17	0.137	14.50	0.090	42	0.175	17.30	0.191
18	0.131	17.70	0.090	43	0.175	17.40	0.162
19	0.140	15.50	0.103	44	0.188	18.50	0.225
20	0.143	18.20	0.127	45	0.188	18.50	0.235
21	0.143	18.50	0.120	46	0.191	17.50	0.197
22	0.143	16.50	0.117	47	0.194	18.50	0.256
23	0.146	15.80	0.100	48	0.197	16.50	0.230
24	0.146	16.00	0.105	49	0.197	16.60	0.210
25	0.150	17.00	0.122	50	0.204	18.60	0.254

A one entry (D) *tariff and a two entries* (D and H) *tariff are wanted. One starts by plotting the data in function of* V *and* D^2 *and in function of* V *and* D^2H *: the numbers plotted are the tree numbers.*


V
0.254
0.241
0.227
0.214
0.201
0.188
0.174
0.161
0.148
0.134
0.121
0.108
0.094
0.081
0.068
0.055
0.041
0.028
0.0091
0.0117
0.0143
0.0169
0.0195
0.0221
0.0247
0.0273
0.0298
0.0324
0.0350
0.0376
0.0402
D²


V
0.254
0.241
0.227
0.214
0.201
0.188
0.174
0.161
0.148
0.134
0.121
0.108
0.094
0.081
0.068
0.055
0.041
0.028
0.118
0.170
0.222
0.275
0.327
0.379
0.432
0.484
0.536
0.589
0.641
0.693
0.746
D²H

The graphs show that the following simple models are valid :

$V = a + bD^2$ *and* $V = a + bD^2H$

Fitting these models by regression requires knowing how the variance of V *varies with* D^2 *and with* D^2H *: the variance of* V *is calculated for some groups of trees chosen such that* D^2 (or D^2H) *is approximately constant in each group.*

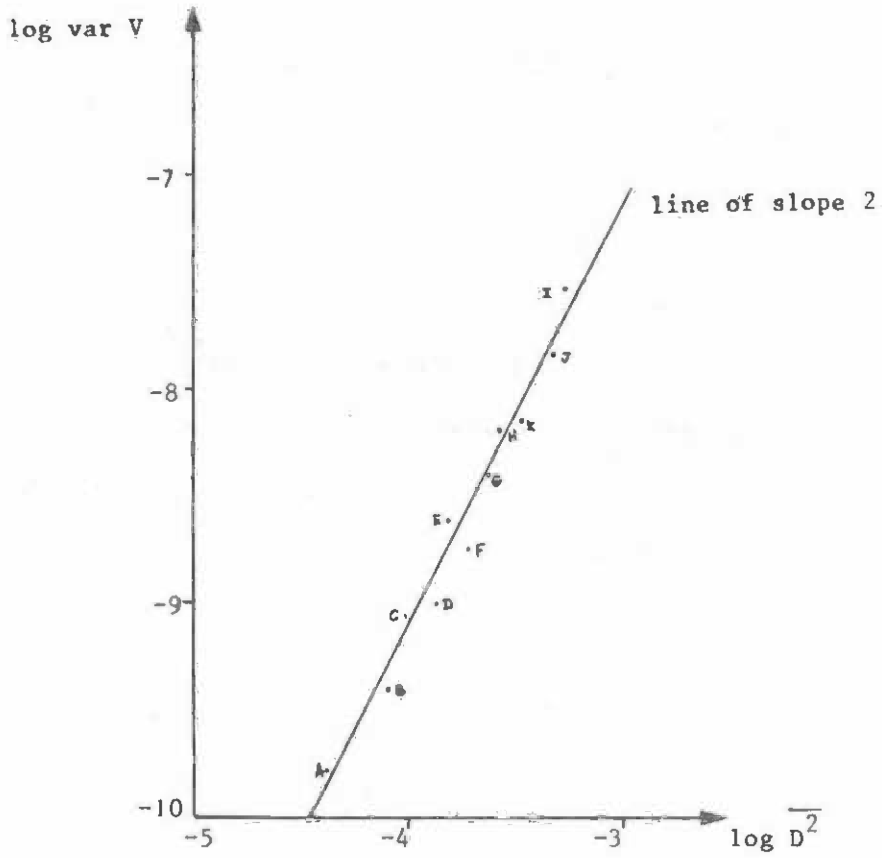
Study of the relationship between var V and D^2

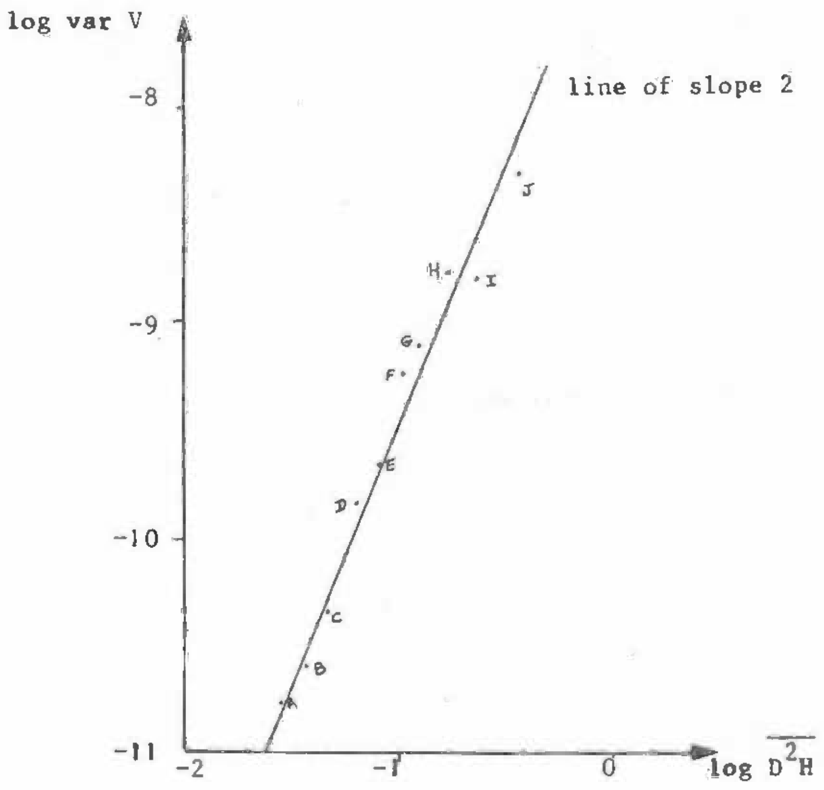
Group n°	Trees n°	$\overline{D^2}$ = mean of D^2	log $\overline{D^2}$	Variance of V = var V	log var V
A	3 - 4 - 5	0.012652	- 4.370	0.0000563	- 9.784
B	9 - 10 - 11 - 12 - 18 -	0.016540	- 4.102	0.0000833	- 9.393
C	13 - 14 - 15 - 16 - 17 -	0.018217	- 4.005	0.0001162	- 9.060
D	20 - 21 - 22 - 23 - 24 -	0.020886	- 3.869	0.000123	- 9.006
E	25 - 26 - 27 -	0.022382	- 3.800	0.0001823	- 8.610
F	29 - 30 - 31 -	0.024327	- 3.716	0.0001580	- 8.751
G	32 - 33 - 34 -	0.026354	- 3.636	0.000225	- 8.399
H	35 - 36 - 37 - 38 -	0.028461	- 3.559	0.0002729	- 8.206
I	47 - 48 - 49 -	0.038532	- 3.257	0.000532	- 7.539
J	44 - 45 - 46 -	0.0356728	- 3.333	0.000388	- 7.855
K	40 - 41 - 42 - 43 -	0.030650	- 3.485	0.0002849	- 8.163

Study of the relationship between var V and D^2H

Group n°	Trees n°	$\overline{D^2H}$=mean of D^2H	log $\overline{D^2H}$	Variance of V = var V	log var V
A	5 - 6 - 10 -	0.215987	- 1.533	0.0000213	- 10.755
B	7 - 8 - 12 -	0.240027	- 1.427	0.0000253	- 10.583
C	9 - 11 - 16 - 17 -	0.266856	- 1.321	0.0000323	- 10.342
D	13 - 14 - 18 - 19 -	0.305533	- 1.186	0.0000537	- 9.833
E	15 - 22 - 23 - 24 -	0.339384	- 1.081	0.0000643	- 9.651
F	20 - 21 - 25 - 26 - 27	0.378446	- 0.972	0.0000983	- 9.227
G	28 - 29 - 30 - 31 -	0.408047	- 0.896	0.000110	- 9.113
H	32 - 33 - 34 - 37 - 38 - 39 -	0.461998	- 0.772	0.000156	- 8.769
I	35 - 41 - 42 - 43 -	0.533143	- 0.629	0.0001529	- 8.786
J	44 - 45 - 46 - 48 - 49	0.646497	- 0.436	0.0002443	- 8.317

Diagrams (log var V, log $\overline{D^2}$) *and* (log var V, log $\overline{D^2H}$) *are given in next page.*


log var V
-7
-8
-9
-10
-5
-4
-3
log $\overline{D^2}$
line of slope 2
A
B
C
D
E
F
G
H
I
J
K


log var V
-8
-9
-10
-11
-2
-1
0
log $\overline{D^2H}$
line of slope 2
A
B
C
D
E
F
G
H
I
J

These graphs show that one can assume that

$$\log \text{var } V = \alpha + 2 \log \overline{D^2}$$

$$\log \text{var } V = \alpha' + 2 \log \overline{D^2 H}$$

Thas it to say :

the variance of V is proportional to $(D^2)^2$

the variance of V is proportional to $(D^2H)^2$.

For the one entry tariff, the weight of tree i *is thus* $w_i = \frac{1}{D_i^4}$; *for the 2 entries tariff, it is* $w_i = \frac{1}{D_i^4 \, H_i^2}$.

RESULTS

One entry tarif : $V = a + bD^2$

a *and* b *are the numbers which minimize the quantity :*

$$S = \sum_{i=1}^{n} w_i \, (V_i - a - bD_i^2)^2 \quad \textit{where} \quad w_i = \frac{1}{D_i^4}$$

If $y_i = \frac{V_i}{D_i^2}$ *and* $x_i = \frac{1}{D_i^2}$, S *can be written :* $S = \sum_{i=1}^{n} (y_i - ax_i - b)^2$

The problem is thus to fit the model y = ax + b *by the usual least squares method : one gets :*

$$a = \frac{\Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} = - 0.02464$$

$$b = \bar{y} - a\bar{x} = 6.5916$$

$$VR \text{ (residual variance)} = \frac{1}{n-2}\left\{\Sigma y^2 - \frac{(\Sigma y)^2}{n} - a\left[\Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}\right]\right\}$$

$$= 0.27501$$

$$\text{var } a = \frac{VR}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} = 0.0000169$$

$$\text{var } b = \frac{VR}{n} + \bar{x}^2 \text{ var } a = 0.04483$$

$$\text{cov }(a,b) = -\bar{x} \text{ var } a = -0.0008153$$

The volume of a stand of N *trees on each of which* D *has been measured will be estimated by :*

$$V_{TOT} = N\,a + b \sum_{i=1}^{N} D_i^2$$

The confidence interval of V_{TOT} *, is, at 0.95 level :*

$$V_{tot} \pm 2\sqrt{\text{var } V_{TOT}}$$

with : $\text{var } V_{TOT} = N^2 \text{ var } a + \alpha^2 \text{ var } b + 2N\alpha \text{ cov}(a,b) + \beta(VR)$

where : $\alpha = \sum_{i=1}^{N} D_i^2$ *and* $\beta = \sum_{i=1}^{N} D_i^4$

Two entries tarif : $V = a + bD^2H$

a *and* b *are the numbers which minimize the quantity :*

$$S = \sum_{i=1}^{n} w_i\,(V_i - a - bD_i^2\,H_i)^2 \text{ where } w_i = \frac{1}{(D_i^2\,H_i)^2}$$

If $y_i = \frac{V_i}{D_i^2\,H_i}$ *and* $x_i = \frac{1}{D_i^2\,H_i}$, S *can be written :*

$$S = \sum_{i=1}^{n} (y_i - ax_i - b)^2$$

Thus, the model $y = ax + b$ *is adjusted by the usual least squares method ; one gets :*

$$a = -\ 0.003609$$
$$b = 0.33677$$
$$\text{var } a = 0.000004922$$
$$\text{var } b = 0.00005569$$
$$\text{cov } (a,b) = -\ 0.0000149$$
$$VR = 0.000533$$

The volume of a stand of N *trees on each of which* D *and* H *have been measured is estimated by :*

$$V_{TOT} = Na + b \sum_{i=1}^{N} D_i^2 H_i$$

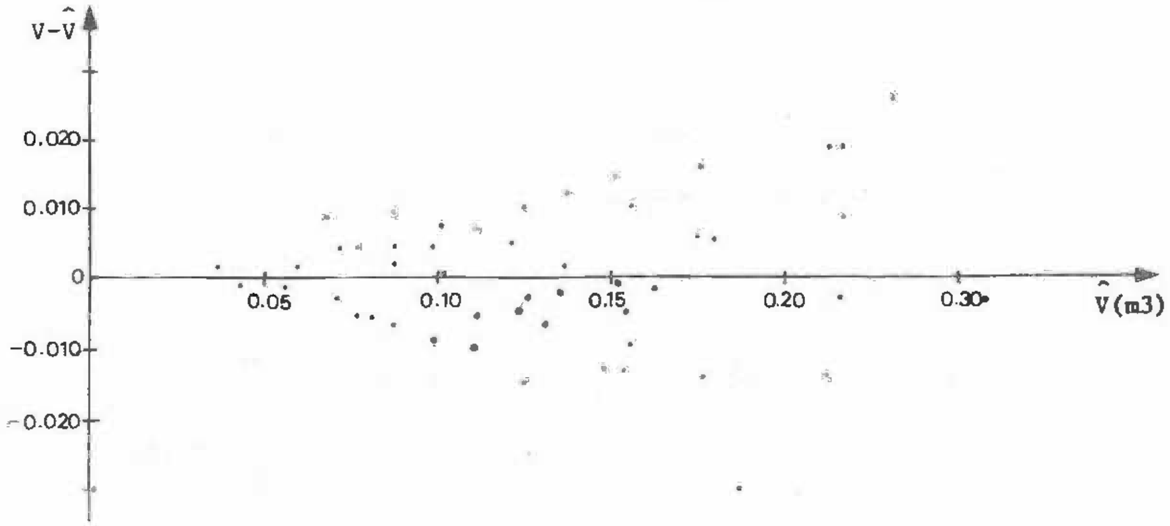
The confidence interval of V_{TOT} *is, at 0.95 level :*

$$V_{TOT} \pm 2 \sqrt{\text{var } V_{TOT}}$$

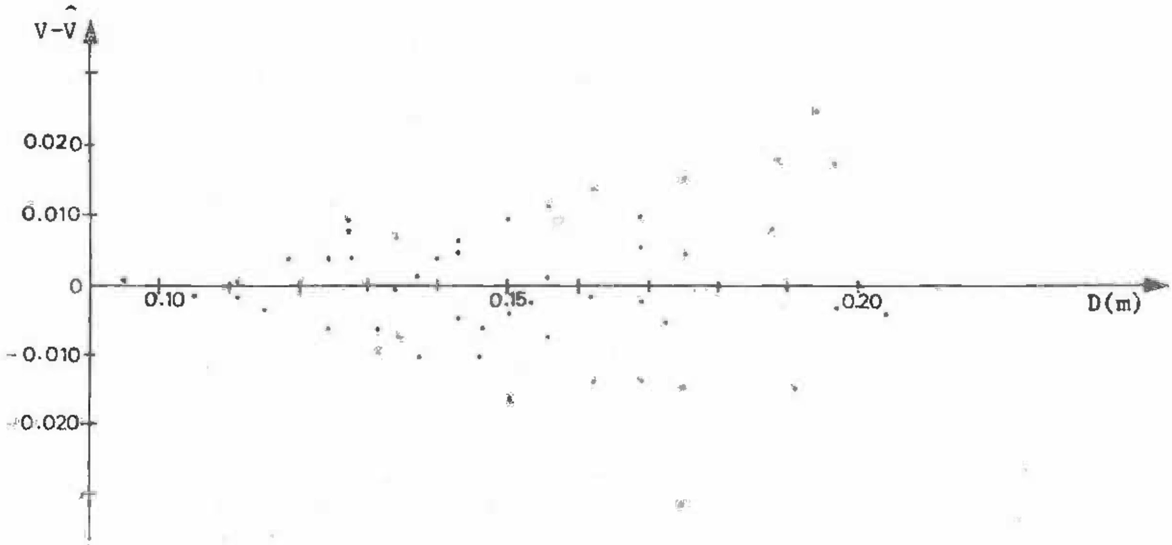
with : $\text{var } V_{TOT} = N^2 \text{ var } a + \alpha^2 \text{ var } b + 2N\alpha \text{ cov } (a,b) + \beta(VR)$

where : $\alpha = \sum_{i=1}^{N} D_i^2 H_i$ *and* $\beta = \sum_{i=1}^{N} D_i^4 H_i^2$.

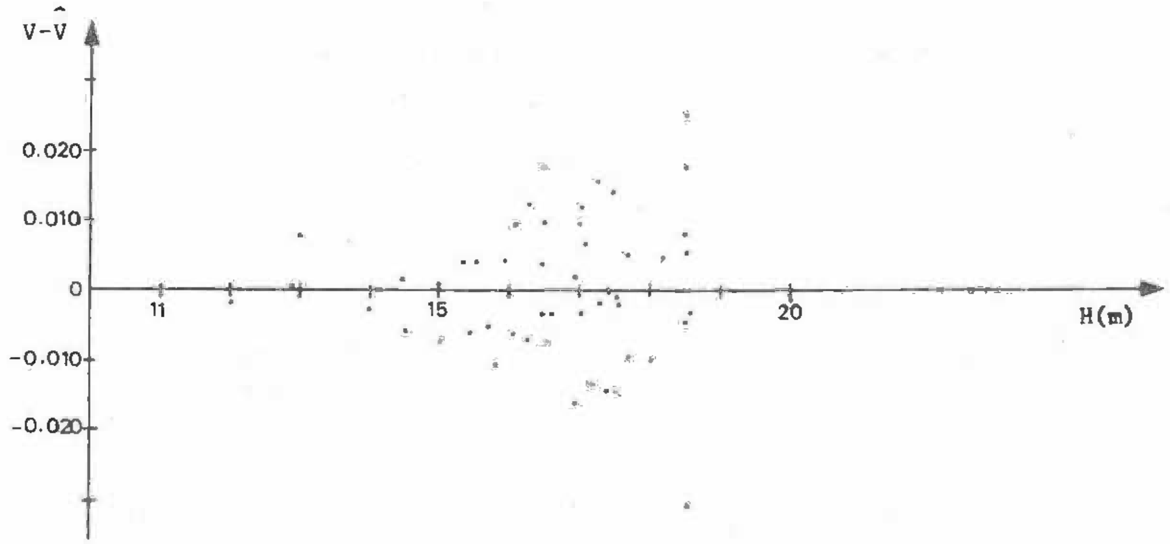
The 3 diagrams of next page concern the 2 entries tariff ; they show that the tariff is correct : the residuals do not tend to vary systematically with $\hat{V}$, D *and* H.



V-V̂
0.020
0.010
0
-0.010
-0.020
0.05
0.10
0.15
0.20
0.30
V̂(m3)



V-V̂
0.020
0.010
0
-0.010
-0.020
0.10
0.15
0.20
D(m)



V-V̂
0.020
0.010
0
-0.010
-0.020
11
15
20
H(m)

36 CONCERNING UNDERBARK VOLUMES

361 Bark thickness and diameter

The bark factor k *is the quotient of overbark diameter over underbark diameter :*

$$k = \frac{D_{ov}}{D_{un}} = 1 + \frac{2B}{D_{un}} = \frac{1}{1 - \frac{2B}{D_{ov}}}$$

D_{ov} = diameter over bark

D_{un} = diameter under bark

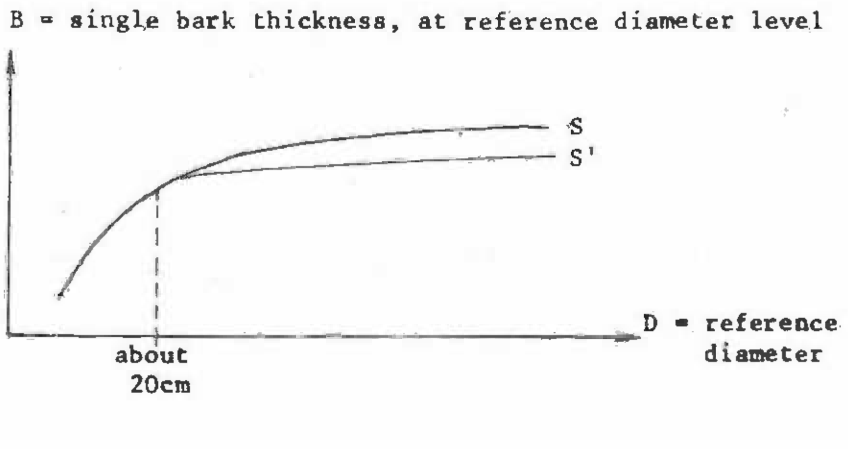
where B *is single bark thickness.*

Bark thickness tends to decrease from the bottom to the tip of the tree but it is not possible to give a general formula for this trend which has to be studied in each case.

k *is sometimes constant from the bottom to the tip of the tree : bark thickness is then proportional to* D_{ov} *(and consequently to* D_{un} *) but it can also happen that, from bottom to tip,* k *decreases first, then remains constant and then increases.*

In general, $\frac{2B}{D_{ov}}$ *varies between 6 and 10 % ;* k *varies therefore between 1.06 and 1.12.*

In most cases (optical measurements on standing trees) only one bark thickness (at reference height) is available per tree. To see if bark thickness at the reference level varies systematically with the reference diameter, start by plotting data :



The cloud of points often has an S *shape : bark thickness increases curvilinearily for small diameters and then more slowly ; sometimes, bark thickness is practically constant for diameters bigger* *than some value (S' curve), but the relationship is seldom strong. This relationship is described by formulas such as :*

(1) $B = a_0 + a_1 D$

(2) $B = \frac{D}{a_0 + a_1 D}$

(3) $B = a_0 D^{a_1} \quad (a_1 < 1)$

B = single bark thickness at reference diameter level

D = reference diameter

Realize that it is a priori incorrect to use such a formula to estimate in a tree bark thickness at different heights, knowing the diameter at these heights because this would give the variables of the formula a meaning different to their definition.

362 Overbark volume - Underbark volume

362.1 Bark quotient *is the quotient of the bark volume over the overbark volume.*

$$P = \frac{V_b}{V_{ov}} = 1 - \frac{V_{un}}{V_{ov}} = \frac{\frac{V_{ov}}{V_{un}} - 1}{\frac{V_{ov}}{V_{un}}}$$

($V_{ov} = V_{un} + V_b$)

V_{ov}: *Volume overbark*; V_{un}: *Volume underbark*; V_b: *volume of bark*

For each volume there is a corresponding bark quotient. The bark quotient related to the bole volume is the most used.

In a given tree, the relationship between P *and* k *depends on the volume which is considered and on the relationship between* k *and height of measurement.*

To get an idea on the form of this relationship, it seems reasonable to suppose that the form factor is the same underbark and overbark, which gives :

$$\frac{V_{ov}}{g_{ov} \times H} = \frac{V_{un}}{g_{un} \times H} \qquad (g = \text{basal area})$$

Thus :

(4) $$P = 1 - \frac{g_{un}}{g_{ov}} = 1 - \frac{1}{k^2} = \frac{2B}{D_{ov}} \left(2 - \frac{2B}{D_{ov}}\right)$$

(Example : $\frac{2B}{D_{ov}} = 8\ \% \rightarrow k = 1{,}087 \rightarrow P = 15{,}4\ \%$)

362.2 Conversion of volume overbark into volume underbark

If overbark and underbark volumes have been measured on a set of trees, a first method is to calculate two tariffs :

$V_{ov} = f\ (D_{ov})$ or $f\ (D_{ov}, H)$

$V_{un} = g\ (D_{ov})$ or $g\ (D_{ov}, H)$

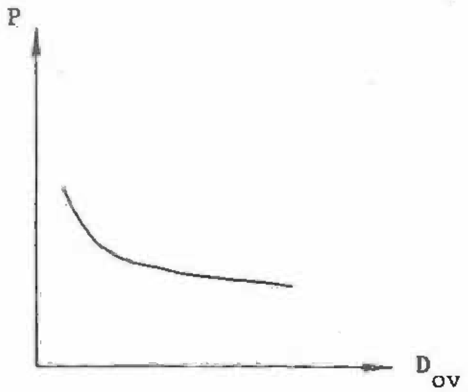
ensuring that V_{un} *is always less than* V_{ov} *because the regression lines can cross each other.*

The following method is more often used :

- *calculate the overbark tariff :*

  $V_{ov} = f\ (D_{ov})$ or $V_{ov} = f\ (D_{ov}, H)$

- *fit a formula giving bark quotient from the tariff's entries. In general, bark quotient is related to reference diameter only :*



The following models

$$P = a_0 + \frac{a_1}{D_{ov}} + \frac{a_2}{D_{ov}^2}$$

(consequence of (1) and (4))

$$P = a_0\ e^{-a_1 D_{ov}}$$

are often adequate.

- *take the following expression for under bark tariff :*

  $V_{un} = (1 - P)\ V_{ov}$

Examples :

$$\left.\begin{aligned} V_{ov} &= a_0 + a_1 D^2 \\ P &= a_2 + \frac{a_3}{D} + \frac{a_4}{D^2} \end{aligned}\right\} \Longrightarrow V_{un} = \left[1 - a_2 - \frac{a_3}{D} - \frac{a_4}{D^2}\right] (a_0 + a_1 D^2)$$

$$\left.\begin{aligned} V_{ov} &= a_0 + a_1 D^2 \\ P &= a_2\, e^{-a_3 D} \end{aligned}\right\} \Longrightarrow V_{un} = \left[1 - a_2 e^{-a_3 D}\right] (a_0 + a_1 D^2)$$

(D is written for D_{ov})

4 ESTIMATION OF USABLE VOLUMES

The measurements which can be done in the field on standing or felled trees provide mainly gross volumes ; supplementary data which can be collected (borings to detect hollows, observation of apparent defects...) can only give indications on useful volumes. The knowledge of usable volumes requires to carry on observations up to the places where the wood is manufactured.

This is an example of procedure followed in tropical high forest (ref. 17) which allows, provided it is carried on completely, to convert bole gross volumes into used volumes.

41 AN EXAMPLE OF METHOD APPLIED IN TROPICAL HIGH FOREST

411 Gathering of data

The procedure is in two phases.

411.1 *In the inventoried region, one makes qualitative observations on standing trees in order to split the bole gross volume in different fractions corresponding each to a quality of standing wood.*

The bole of each tree is virtually divided into 3 parts of equal length and each part receives three notes (these notes range from 1 to 5 - see next page) which describe respectively : the form of the bole, its health and the appearance of the wood. The three notes given to each third of bole are pooled in one note ranging from 1 to 5 according to the following correspondence grid :

Notes given to the third of bole			Pooled note given to the third of bole
Form (F)	Health (H)	Appearance of the wood (W)	
1	1	1	1
1	1	2	
2	1	1	
2	1	2	
1	2	1	2
2	2	1	
1	2	2	
2	2	2	
1	1	3	
1	2	3	
2	1	3	
2	2	3	
3	1	1	
3	1	2	
3	2	1	
3	1	3	
3	2	2	
3	2	3	

Notes given to the third of bole			Pooled note given to the third of bole
Form (F)	Health (H)	Appearance of the wood (W)	
1	3	1	3
1	3	2	
2	3	1	
1	3	3	
3	3	1	
2	3	2	
2	3	3	
3	3	2	
3	3	3	
a 4 in the third column			
Every set with one or several 4 (except one 4 in the third column)			4
Every set with one or several 5			5

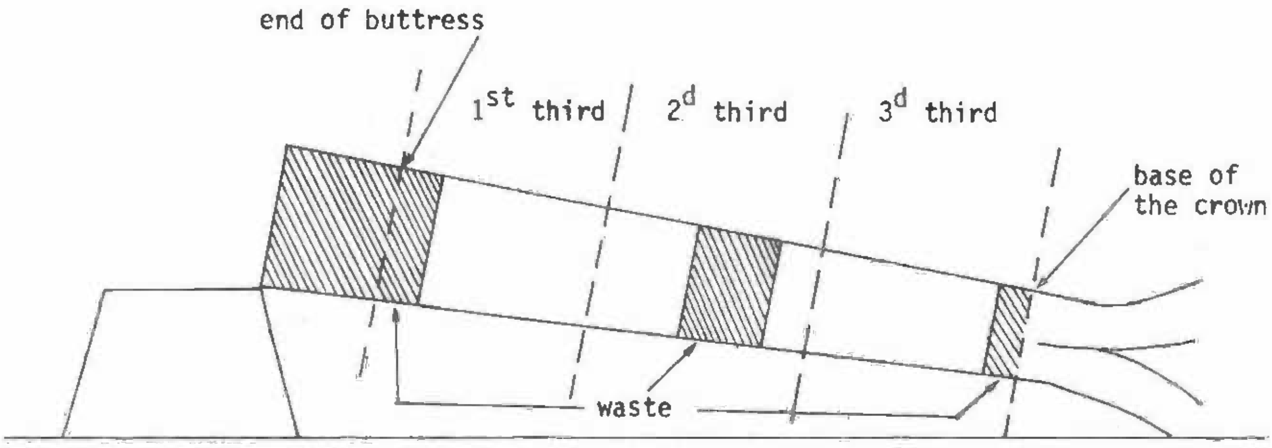
VALUE OF THE QUALIFICATIONS

Qualifications →	1	2	3	4	5
FORM (F)	Straight and Cylindrical	1 slight bend ------ Conical shape ------ Oval section ------ 1 flat surface over the whole length 1 slight groove ------ 2 or 3 flat surfaces above the buttresses ------	1 pronounced bend ------ Conical shape + Oval section ------ 2 or 3 flat surfaces 2 slight bends ------ 1 extended buttress ------ 2 slight grooves ------	1 pronounced bend +1 extended buttress or + 1 groove 2 m or + 2 or 3 flat surfaces ------ 1 extended buttress +1 groove 2 m or +2 or 3 flat surfaces ------ 1 groove 2 m +2 or 3 flat surfaces 2 bends (pronounced) ------ 1 pronounced bend +1 slight bend 1 rib	fluted section ribbed section (2 ribs or more) ------ 1 elbow ------ 1 "bayonet" ------ 1 deep groove of 2 meters ------
HEALTH (H)	Sound (neither off-shoots nor covered knots)	1 large off-shoot ------	2 large off-shoots ------ 1 black trace	More than 2 large off-shoots 1 broken branch 1 woodpecker hole	Visible decay at foot ------ 1 decayed knot Hollow sounding stem
APPEARANCE OF THE WOOD (W)	Straight grain and no defect (no thorns, splinters, "grains d'orge", wound marks, excres-cences, etc...)	Irregular grain (very slight ribs in all directions) ------ 1 healed wound mark ------ 1 slight swelling ------ Visible thorns ------ Slight localised twisting ------ Raised bark in many places	Slight twisting<15° ------ 2 or 3 swellings or large healed knots various wound marks	Slight twisting <15° + slight swelling ------ More than 3 swellings in large healed knots	Spiral grain >15° ------ Swelled surface ------

On an average, the bole volume is distributed amongst the height thirds according to the fractions : 44% for the bottom third, 33% for the middle third, 23% for the upper third. With these figures, the bole gross volume can be distributed amongst the 5 classes of apparent quality.

411.2 *Being determined from external defects visible on standing trees, the 5 classes which result from the first phase give only an approximate evaluation of wood quality. A second phase takes place on logging companies near the inventoried region : the qualitative observations are made on trees before felling ; after felling, the evolution of each third of log is followed :*

a/ *Measurement of the parts eventually left in forest before tractor logging : stump-waste, waste due to pollarding, waste in the central part of the bole to eliminate a big defect, if any.*



b/ *measurement of waste left on loading area after logging.*

c/ *if possible, measurement of parts left on the sawmill park or before the embarkation in case of export.*

It is easy to realize that these operations are difficult and require full time employees during several months. Operation c/ is often impossible.

412 Analysis of data

412.1 *If only operations* a/ *and* b/ *have been done, it is possible to estimate, for each apparent quality class, the proportion of volume which comes out of forest.*

Example : SIBITI ZANAGA region (République Populaire du Congo). Okoumé (Aucoumea klaineana) exploited for veneer in Pointe Noire : 76 observed trees.

Apparent quality	1	2	3	4	5	Total
Distribution of bole volume of the 76 trees	42.9%	35.1%	12.9%	1.7%	7.4%	100%
Proportion of volume coming out of forest	73.6%	56.7%	10.6%	9 %	25.8%	
Volume coming out of forest / Standing bole gross volume, all qualities together	31.6%	20 %	1.4%	0.2%	1.9%	55%

0.429×0.736

55% is the global merchantable coefficient : it represents the ratio :

$$\frac{\text{volume coming out of the forest}}{\text{standing volume}}$$

Suppose that qualitative observations on N *trees during the inventory of a forest give the following figures :*

Apparent quality	1	2	3	4	5	Total
Distribution of bole volume of the N trees	50 %	30 %	8 %	2 %	10 %	100%

If this forest is to be exploited in the same purpose and in the same way, its merchantable coefficient can be estimated by :

(0.50 × 0.736) + ... + (0.10 × 0.258) = 57.4 %.

412.2 *If operation* c/ *is done, it is possible to share this volume between different categories (log-export, local sawing,..).*

42 ESTIMATION OF THE USABLE VOLUME BY A TARIFF

The direct procedure is : fell a sample of trees, measure the useful volume V_u *and construct a tariff* $V_u = f(D)$ *or* $V_u = f(D,H)$.

This can rarely be done because the necessary sample size is far larger than for a tariff giving the gross volume because the variability of internal defects adds to the variability of tree forms.

An indirect procedure is therefore followed in practice :

(a) *with a sample of trees, construct a tariff giving the gross volume* V.

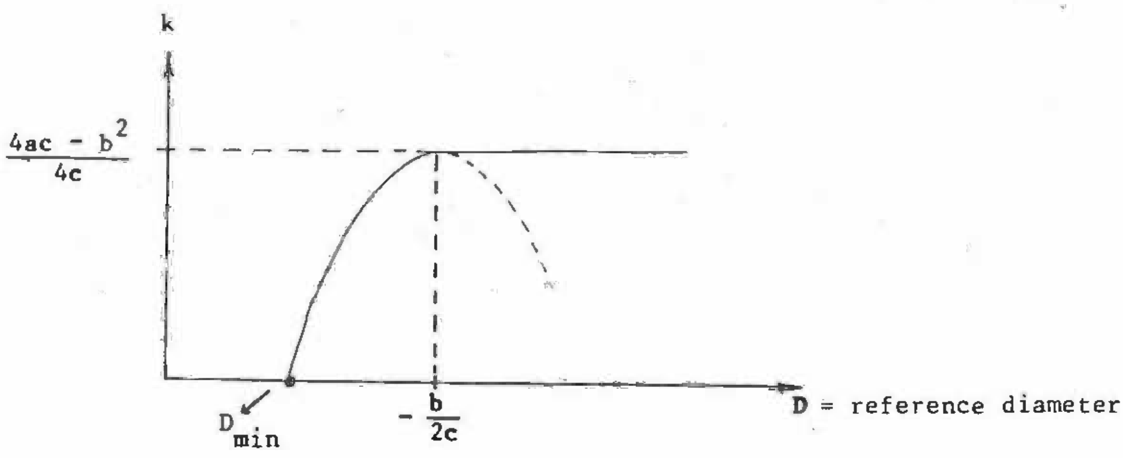
(b) *fell a set of trees (belonging if possible to the sample), measure the gross volume* V *and the usable volume* V_u *and calculate for each tree the ratio :* $k = \frac{V_u}{V}$

(c) *with these ratios, fit a model giving* k *in function of tariff entries ; in general a function of* D *alone is taken as model.*

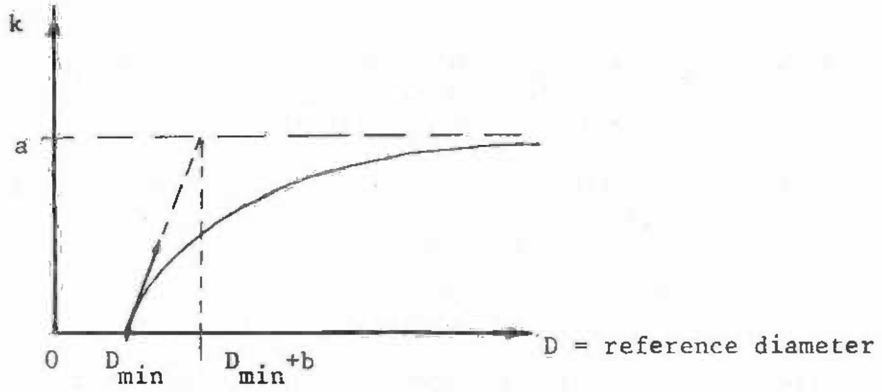
1st example : $k = a + bD + cD^2$ *; this parabola starts from a point where the reference diameter equals the minimum diameter of a merchantable log,* $D = D_{min}$

$$D_{min} = \frac{\sqrt{b^2 - 4ac} - b}{2c}$$

For $D = -\frac{b}{2c}$ *, the parabola reaches its maximum. The descending part of the curve is not used ; it is replaced by the horizontal line.*



<u>*2nd example*</u> : $k = a\left(1 - e^{\frac{D_{min} - D}{b}}\right)$



(d) *take for* "usable volume-tariff" $V_u = kV$ *where* V *is the function established in* (a) *and* k *the function established in* (c).

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-:-:-:-:-:-:-:-:-:-:-:-:-:-:

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